



CUDA and Fermi Optimization Techniques

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Agenda



- **CUDA 101 (General Overview)**
- **Toy of FDM example**
- **Monte Carlo Simulation & Time Series Analysis**
- **General CUDA Optimization Tips**

CUDA 101

General Overview



CUDA

**Graphics
(GPU)**

**Parallel
Programming**

CUDA Parallel programming



- **GPU Knowledge**

- Cg
- OpenGL
- DirectX

- **Parallel knowledge**

- Pthreads / winthreads
- MMX, SSE
- OpenMP
- PVM / MPI

- **Heterogeneous Knowledge**

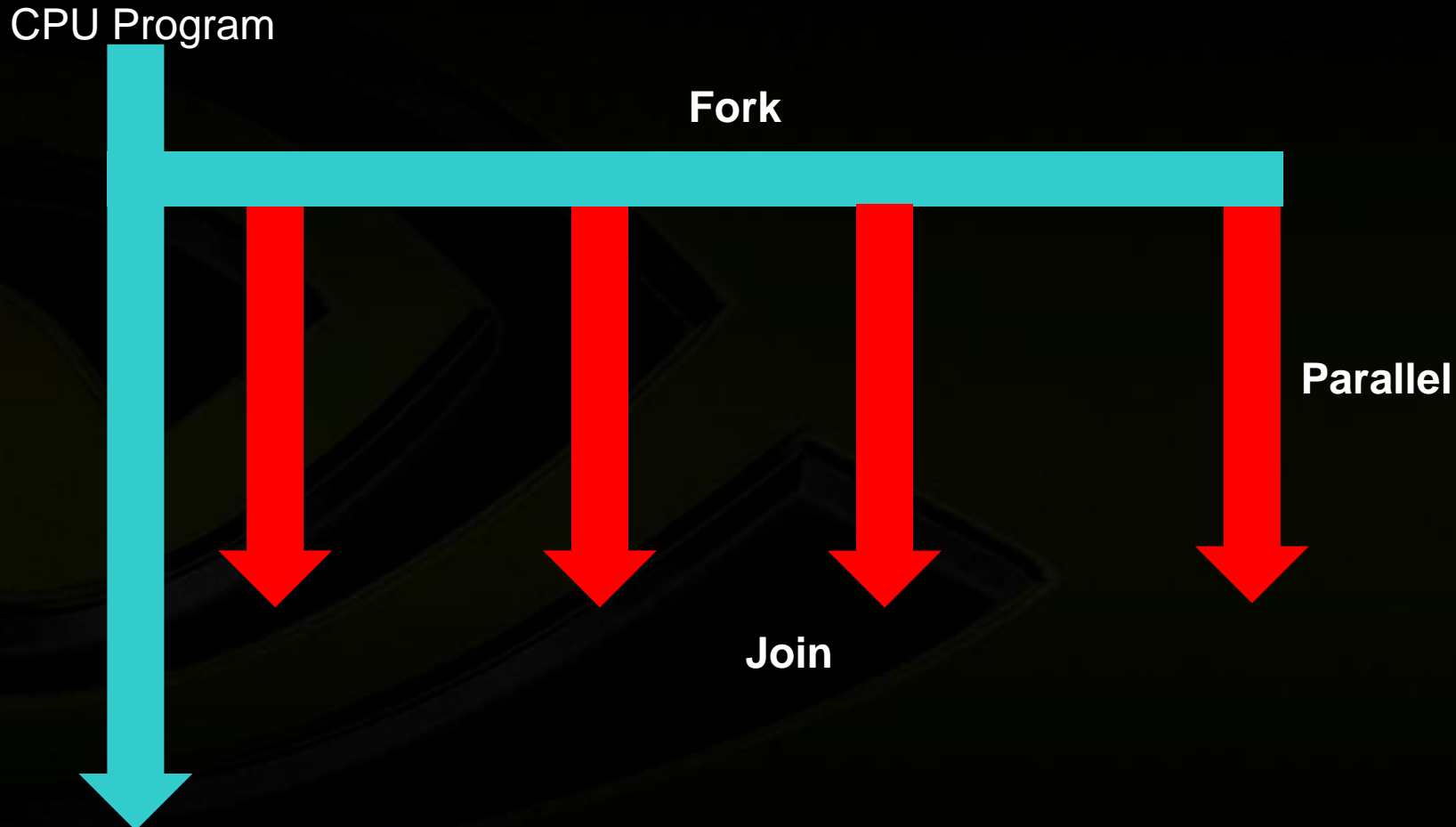
- GPU
- Parallel DSP
- Parallel ASIC
- Parallel FPGA
- Cell BE



CUDA

Parallel Computing with GPU

Parallel Model in OpenMP



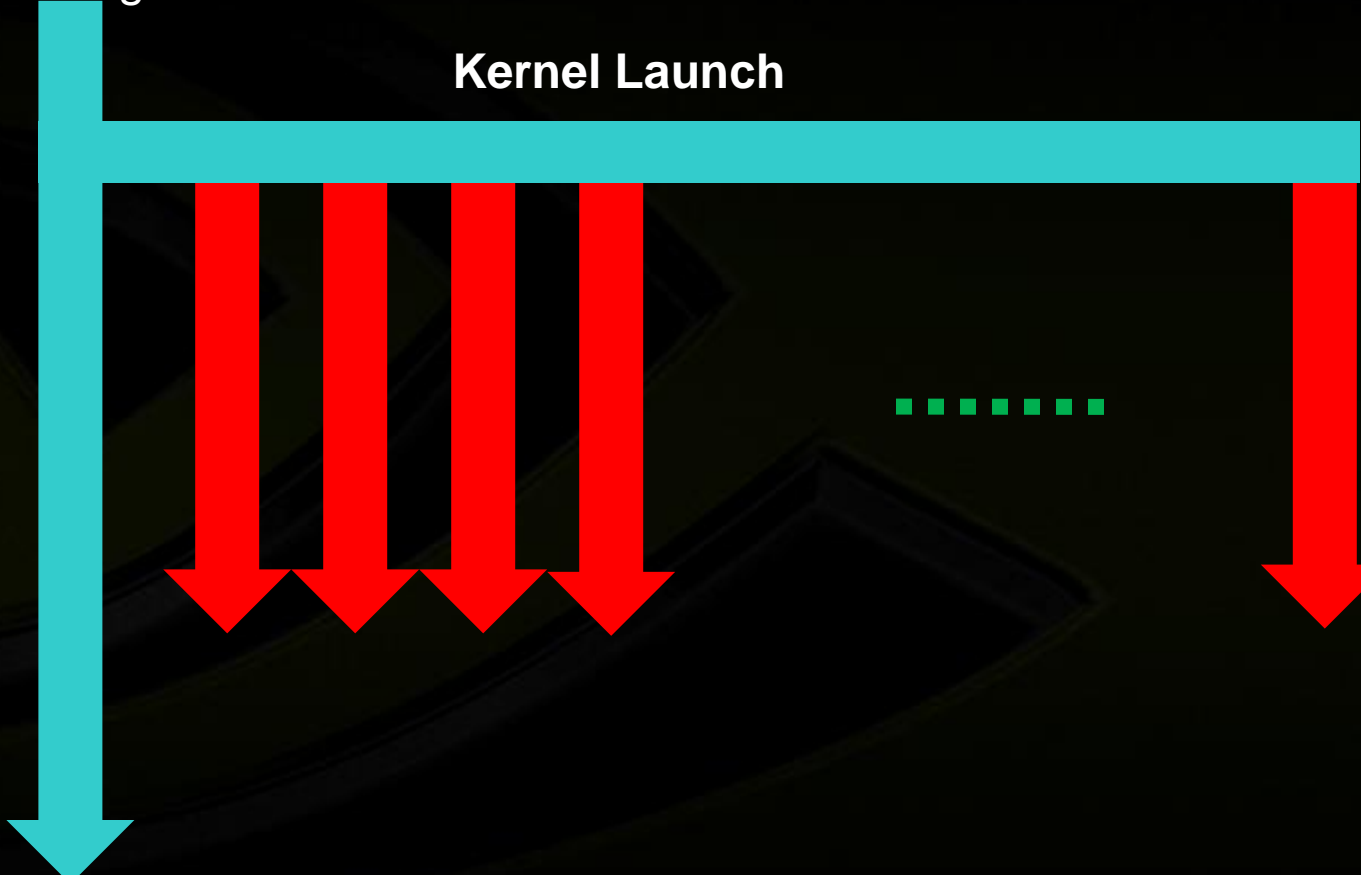
CUDA parallel Model



CPU Program

Kernel Launch

GPU threads



Saxpy Example : CPU serial



```
for (int i = 0; i < n; ++i) {  
    y[i] = a*x[i] + y[i];  
}
```

i

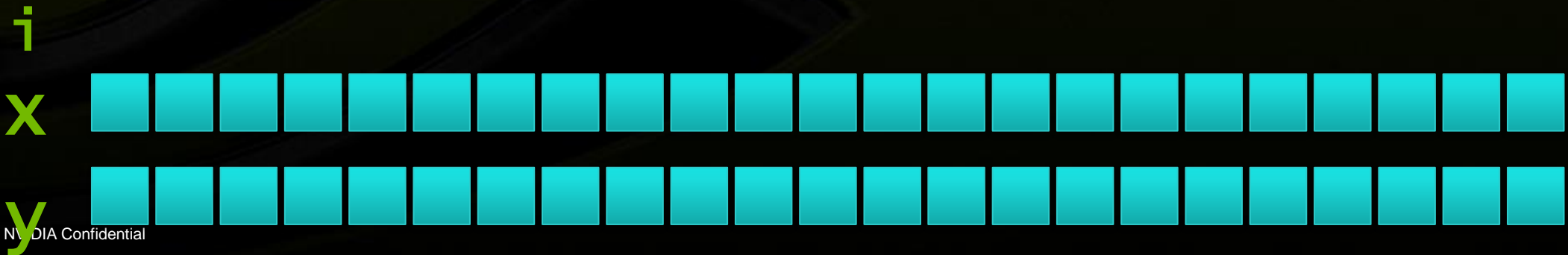
x

y



Example of Saxpy Parallel : OpenMP

```
# pragma omp parallel shared (n,a,x,y) private (i)
# pragma omp for
for (int i = 0; i < n; ++i) {
    y[i] = a*x[i] + y[i];
}
```



Example of Saxpy Parallel : MPI

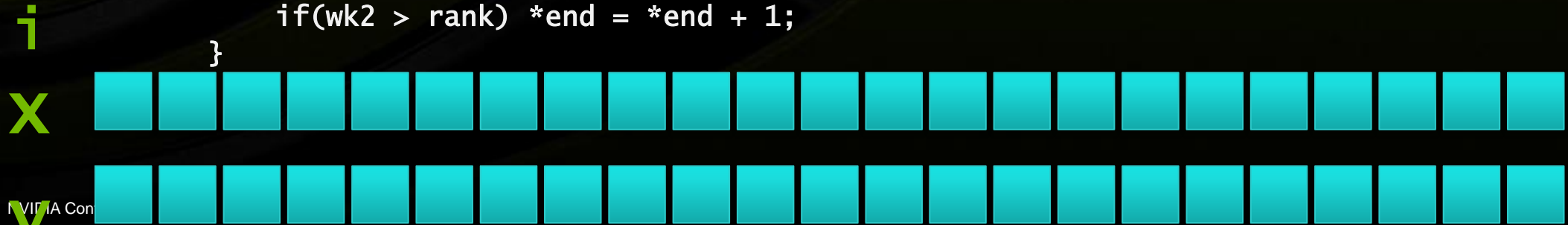
```

for ( i = start ; i < end; i++)
{
y[i] = a * x[i] + y[i];
}

MPI_Init(&argc, &argv);
MPI_Comm_rank(MPI_COMM_WORLD,&rank);
MPI_Comm_size(MPI_COMM_WORLD,&size);

void para_range(int lowest, int highest,int nprocs, int myrank,
                int *start, int *end) {
    int wk1, wk2;
    wk1 = (highest - lowest + 1) / nprocs;
    wk2 = (highest - lowest + 1) % nprocs;
    *start = myrank * wk1 + lowest + ( (rank<wk2) ? myrank : wk2);
    *end = *start + wk1 - 1;
    if(wk2 > rank) *end = *end + 1;
}

```





Example of Saxpy Parallel : SSE

```
void saxpy_vector(short *z, short *x, short *y, short a, unsigned n) {  
    __m128i* x_ptr = (__m128i*) x;  
    __m128i* y_ptr = (__m128i*) y;  
    __m128i* z_ptr = (__m128i*) z;  
    __m128i a_vec = _mm_splat_epi16(a);  
    int i;  
    for (i = 0; i < n/8; ++i) {  
        __m128i x_vec = x_ptr[i];  
        __m128i y_vec = y_ptr[i];  
        __m128i z_vec = _mm_add_epi16( _mm_mullo_epi16 x_vec, a_vec), y_vec);  
        z_ptr[i] = z_vec;  
    }  
}
```

i

X



NVIDIA Con

Y



Saxpy Parallel : CUDA



```
{  
x[i] = a * x[i] + t * y[i];  
}
```

```
Saxpy <<<N ,M >>> (n, 2.0, x, y);
```

i

x



y



CUDA C extension

Launch the kernel

Function <<< Grid, Block >>> (parameter);

Additional C standard API for mem control

cudaXXX : cudaMalloc, cudaMemcpy,

cuXXX : cuMalloc, cuMemcpy

cutXXX : cutXXX

For Function

__global__, __device__, __host__, __device__ __host__

For memory

__shared__, __device__, reg/loc

pre-defined variables

blockDim, blockIdx, threadIdx, cudaMemcpyHostToDevice

Pre-defined function

__syncthreads(), __mul24(); etc

Process of CUDA developing



Serial

Algorithm
serial Programming
Compile
Debugging
Release



CUDA parallel

Algorithm
serial Programming
Compile
CUDA convert
Profile
Parallelize
Compile
Debugging
Optimize/profile
Release

CUDA is Parallel Computing !!!



Serial

Algorithm
serial Programming
Compile
Debugging
Release

MPI parallel

Algorithm
serial Programming
Compile
parallel Programming
Profile
Parallelize
Compile
Debugging [totalview]
Optimize
Release
MPIrun

CUDA parallel

Algorithm
serial Programming
Compile
CUDA convert
Profile
Parallelize
Compile
Debugging
Optimize/profile
Release

CPU Program

Kernel Launch

SM

Block

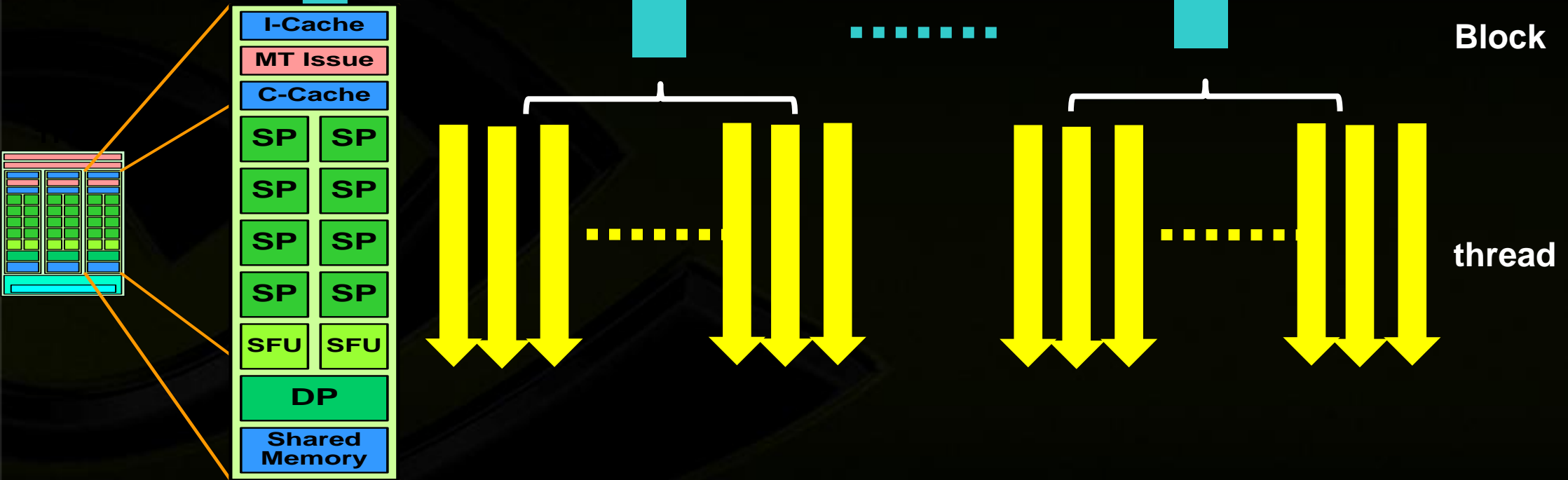


Image Processing Diagram without CUDA

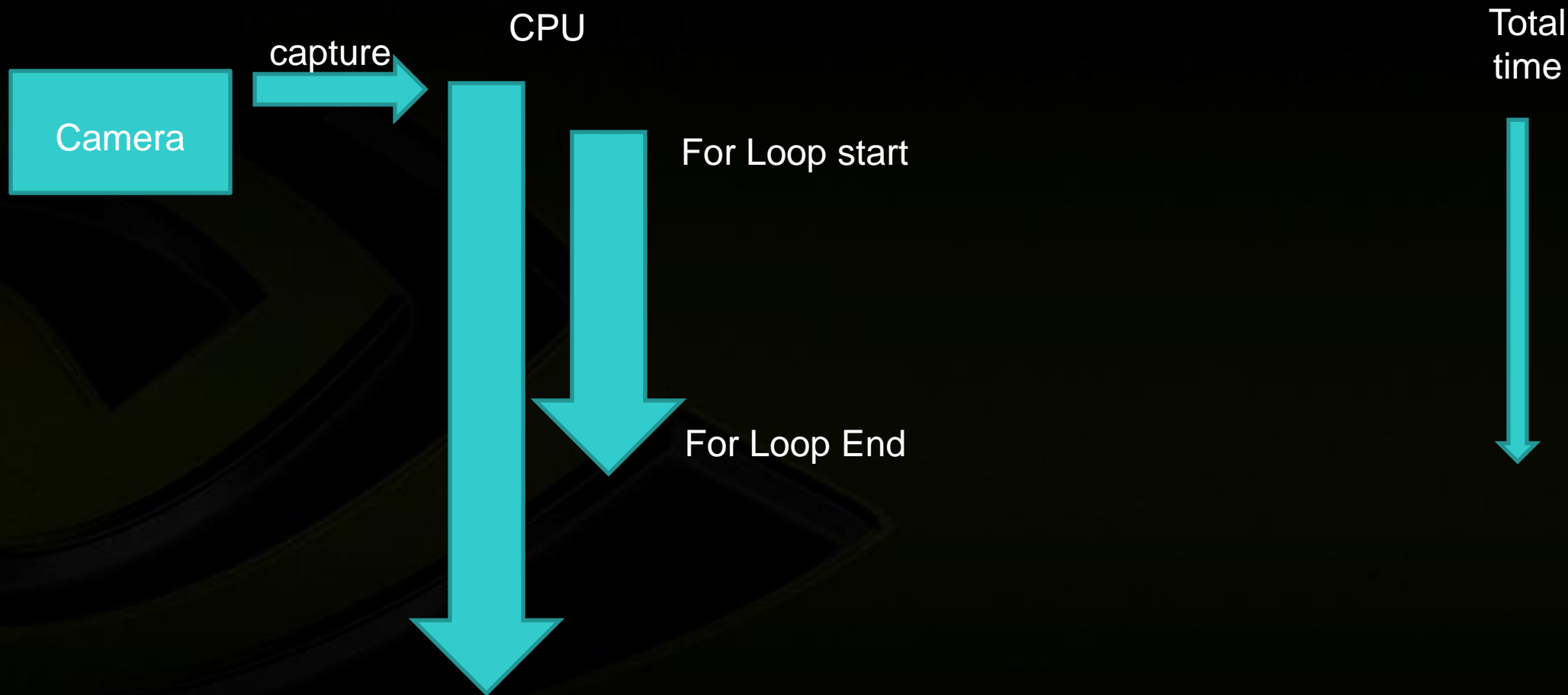
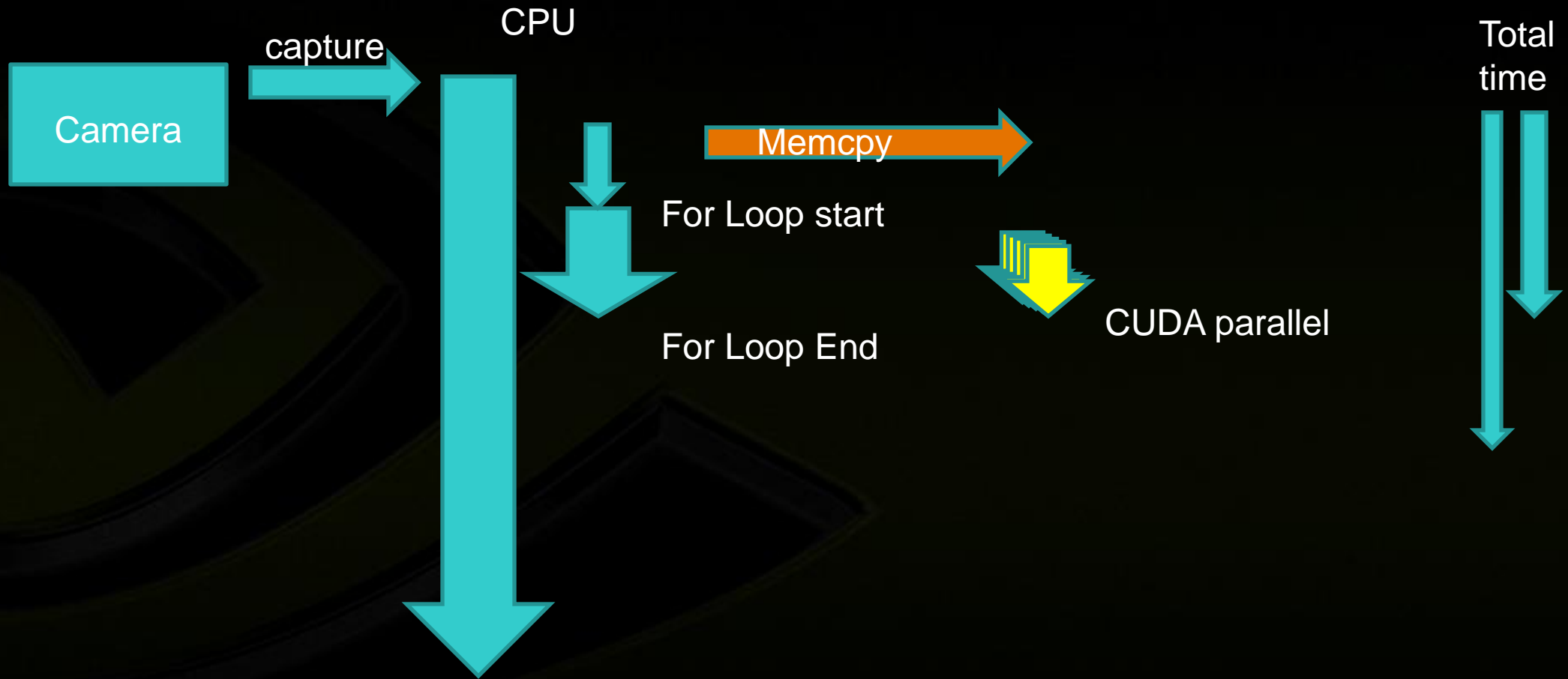


Image Processing Diagram with CUDA



1D Heat Equation

CUDA Toy
for undergraduate student



1D Heat Equation



A diagram illustrating a 1D heat conduction problem. It shows a vertical cyan cylindrical bar standing on a red cylindrical base. A white curly bracket on the right side of the cyan bar spans its entire height and is labeled "1D bar". The red base is labeled "Heat Source".

1D bar

Heat Source

1D Heat Equation



$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2}$$

1D Heat Equation

$$u(0, t) = u(1, t) = 0$$

Boundary condition

$$u(x, 0) = u_0$$

Initial condition

Discretization



$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2}$$

↓ discretization

$$\frac{u_{j,i+1} - u_{j,i}}{\Delta t} = \alpha \frac{u_{j+1,i} - 2u_{j,i} + u_{j-1,i}}{\Delta x^2}$$

Forward difference with second order
i(time), j(space)

↓ relation

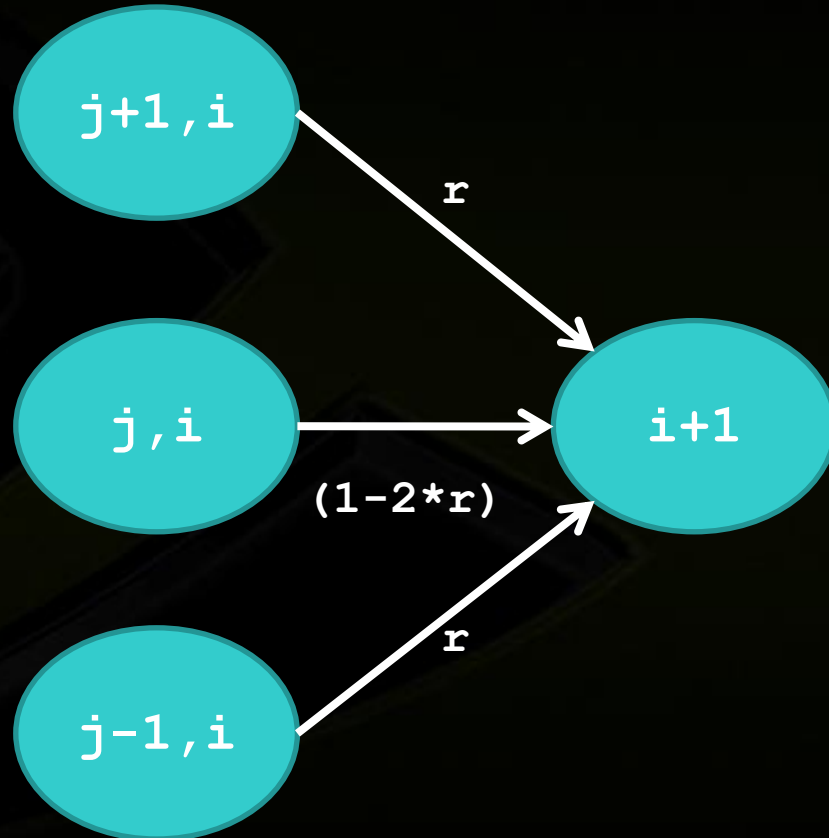
$$u_{j,i+1} = ru_{j-1,i} + (1 - 2r)u_{j,i} + ru_{j+1,i}$$
$$r = \alpha \Delta t / \Delta x^2$$

Explicit method

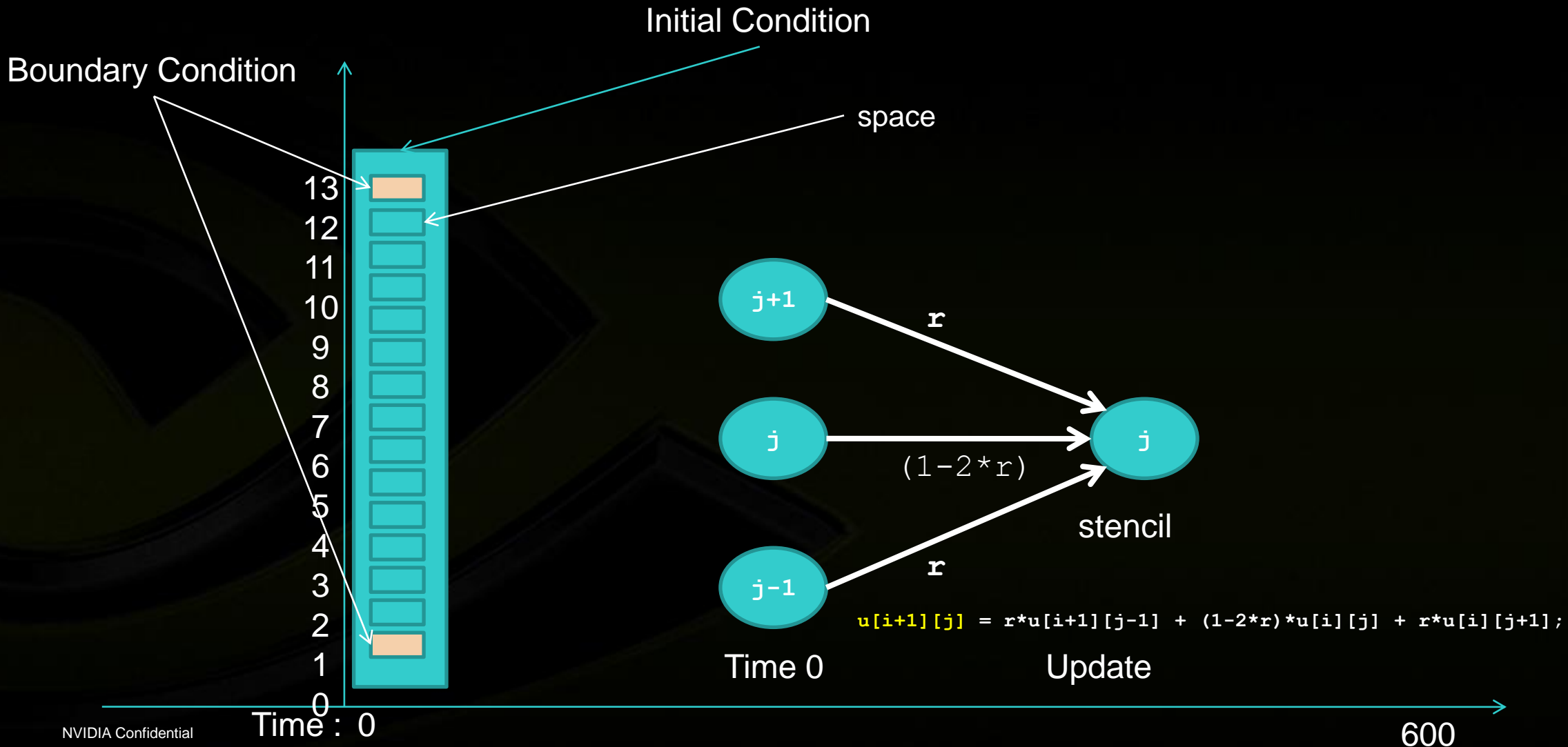
```
u[i+1][j] = r*u[i+1][j-1] + (1-2*r)*u[i][j] + r*u[i][j+1];
```

Discretization (stencil)

$$u_{j,i+1} = ru_{j-1,i} + (1-2r)u_{j,i} + ru_{j+1,i}$$



Conceptual Diagram



Explicit Pseudo Code

- **Parameter and data Initialization**
 - Stencil, boundary/initial condition,

- **FOR LOOP (time, i)**

FOR LOOP (stencil, j)

Update the stencil relation

$$u[i+1][j] = r*u[i+1][j-1] + (1-2*r)*u[i][j] + r*u[i][j+1];$$

- **Results**

CPU code



- **$u[i][j]$ vs. $u[j]$**
 - **$u[i][j]$ easy to develop**
 - Possible to visualize the process
 - **$u[j]$ efficient to use memory**
 - Get the last result

Main algorithm

```
for( i =0; i < N; i++) {
```

← Time Iteration

```
    for( j =0; j < M; j++) {
```

← Space Iteration

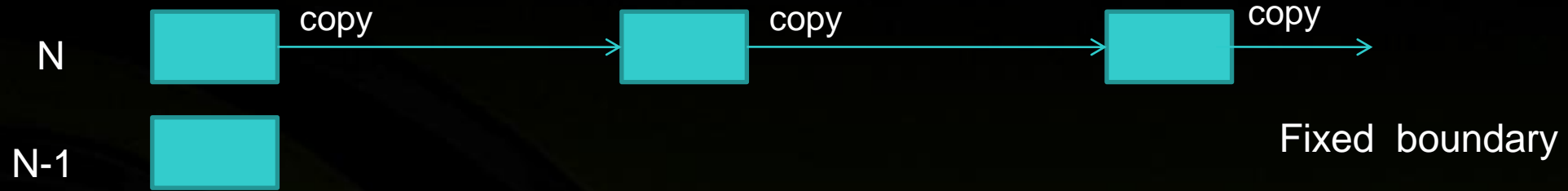
$$u[i+1][j] = r * u[i+1][j-1] + (1-2*r) * u[i][j] + r * u[i][j+1];$$

Heat relation

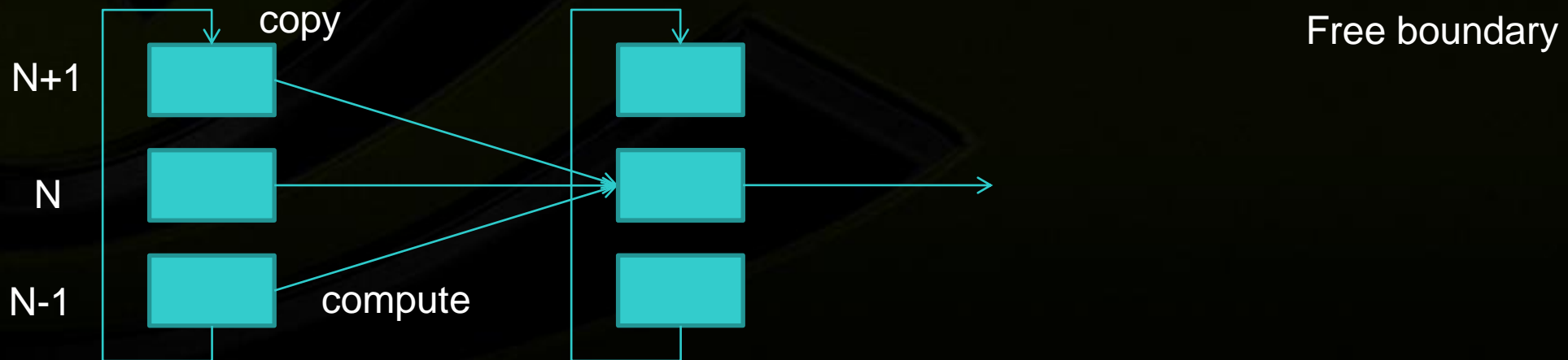
Boundary Condition



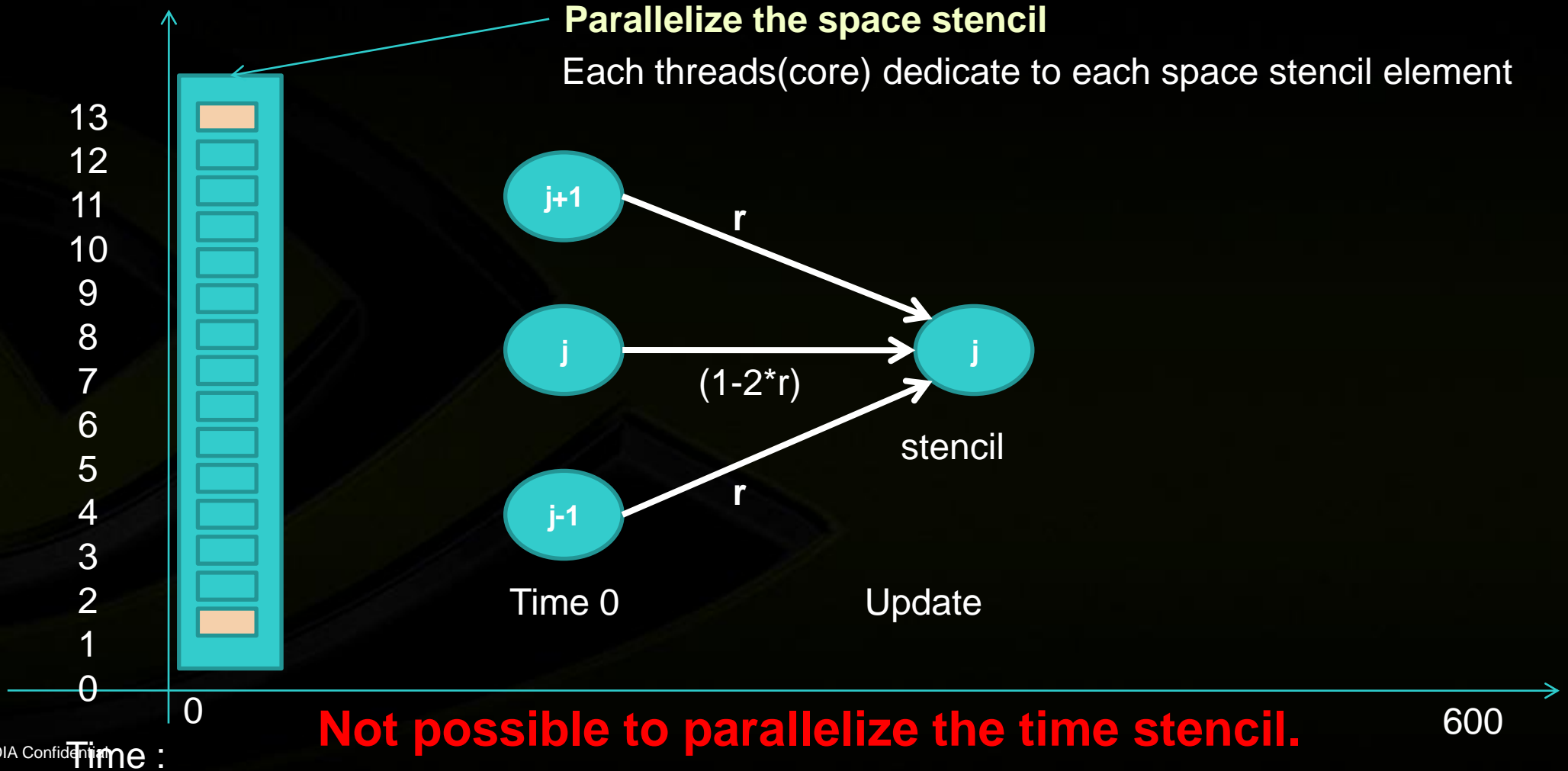
Method1



Method2



How to Parallelize

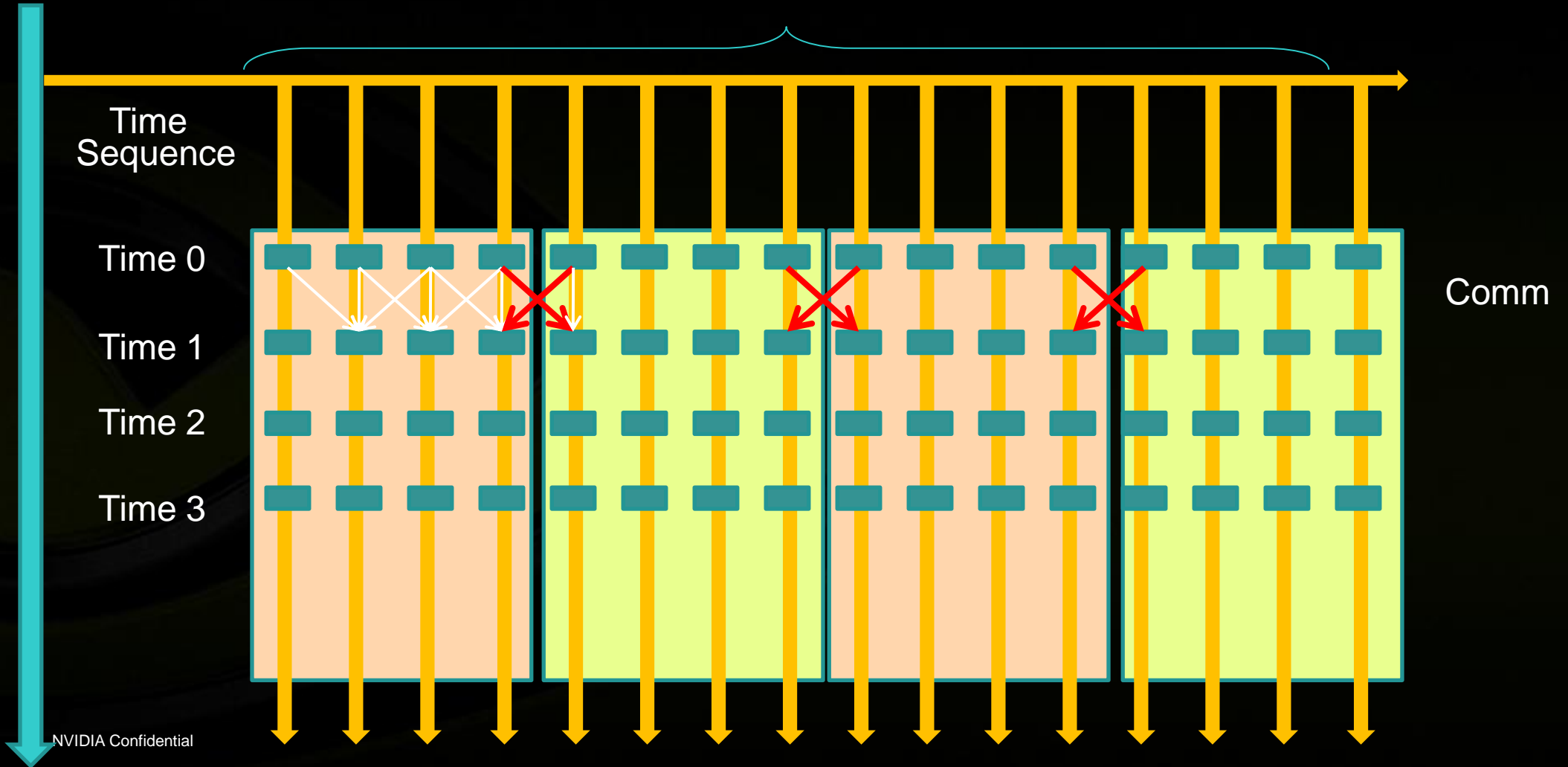


How to parallelize

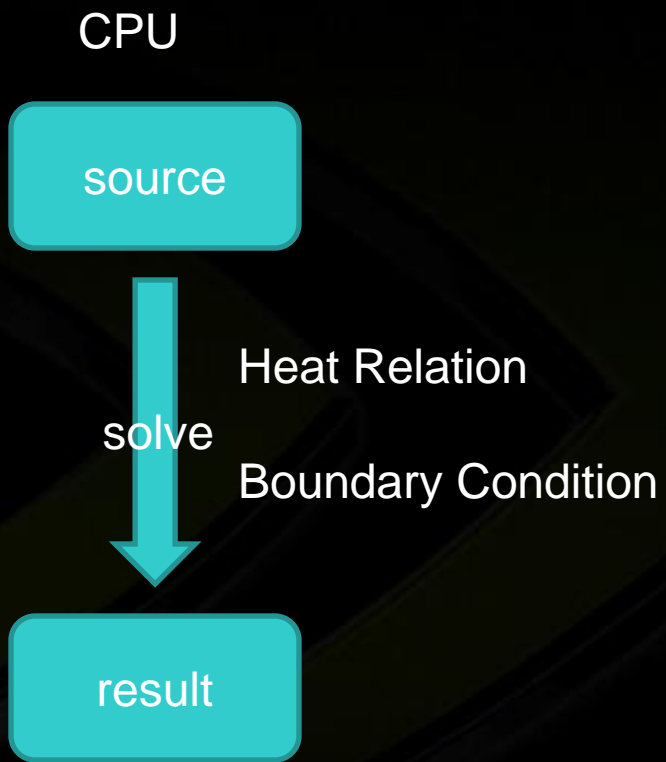


- **Parameter and data Initialization**
 - Stencil, boundary/initial condition,
- **FOR LOOP (time, i)**
 - FOR LOOP (stencil, j)**
 - Update the stencil relation**
- **Results**

Space parallelization



CPUcode



CPU code

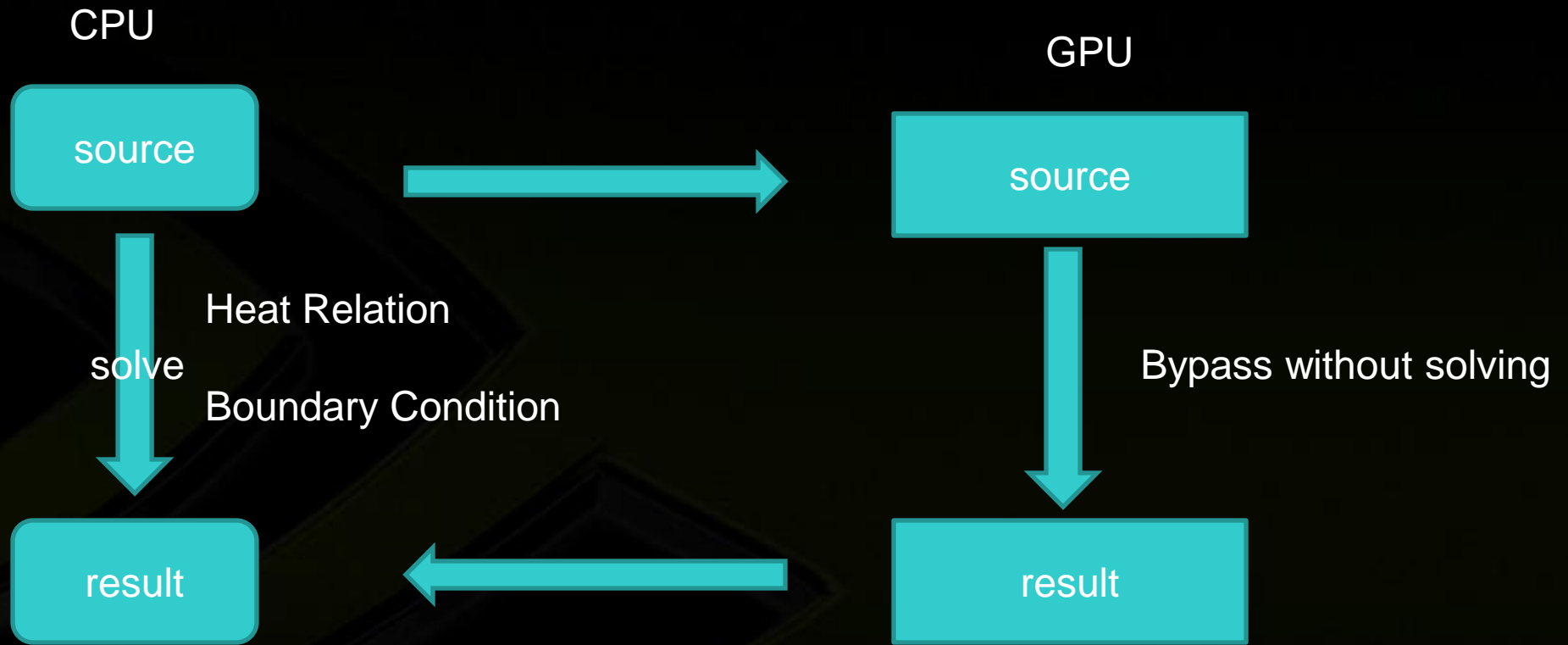


```
do{
    time += dt; printf("aaaaaa %f\n",time);
    for(i=1; i < mesh+1; i++){
        temper[i].new_1 = (double) (1-2*r)*temper[i].old + r*(temper[i-1].old + temper[i+1].old);
        printf("\n processing \t %d %f %f \n",i, temper[i].new_1, temper[i].old);
    }
    temper[mesh+1].new_1 = temper[mesh-1].new_1;
    printf("\t print results %d %f %f \n",mesh+1, temper[mesh+1].new_1,temper[mesh-1].new_1 );

    for(i=1; 1 < mesh+2; i++)
temper[i].old = temper[i].new_1; printf("aa\t\t %d %f %f \n",i, temper[i].new_1,temper[i].new_1 );
    if(++count % print_step)==0){
    printf("hh \t\t\t %10.5lf", time);
    for(i=0; i<mesh; i+=2)
        printf("df 8.4lf", temper[i].new_1);
    if(!(i%2))
        printf("fd %8.4lf\n", temper[mesh].new_1);
    }

    printf("\n\n time print %f\n", time); getchar();
}while(time < end_time);
    if((count % print_step)){
        printf("bghj %10.5lf", time);
        for(i=0; i<mesh; i+=2)
            printf("ahg 8.4lf", temper[i].new_1);
        printf("nhgf %8.4lf\n", temper[mesh].new_1);
    }
}
```

GPUcode-01 Memory Map



GPUcode-01 Malloc Template



```
double* init_GPU_data(struct flow * temper, int mesh)
{
    double *u_dev; // for GPU data upload
    size_t gpuMemsize=sizeof(double)*(mesh+2) ;
    double *u_host; // temperal value
    cudaMalloc( (void**)&u_dev, gpuMemsize);cudaErr("malloc u_dev");
    u_host = (double *) malloc( gpuMemsize);
    for(int i=0;i<mesh;i++){
        u_host[i]= temper[i].old;
        printf("before %d : data initial :u_host[%d]= %f temper[%d].old  =%f\n", i, i, u_host[i], i,
temper[i].old);
    }
    cudaMemcpy(u_dev, u_host, gpuMemsize, cudaMemcpyHostToDevice);cudaErr("memcpy u_dev u_host");
    cudaMemcpy(u_host, u_dev, gpuMemsize, cudaMemcpyDeviceToHost);cudaErr("memcpy u_dev u_host");
    for(int i=0;i<mesh;i++){
        printf("after %d : data initial :u_host[%d]= %f temper[%d].old  =%f\n", i, i, u_host[i], i,
temper[i].old);
    }

    free(u_host);

    return (double *)u_dev;
}
```

GPUcode-01 Launch Template

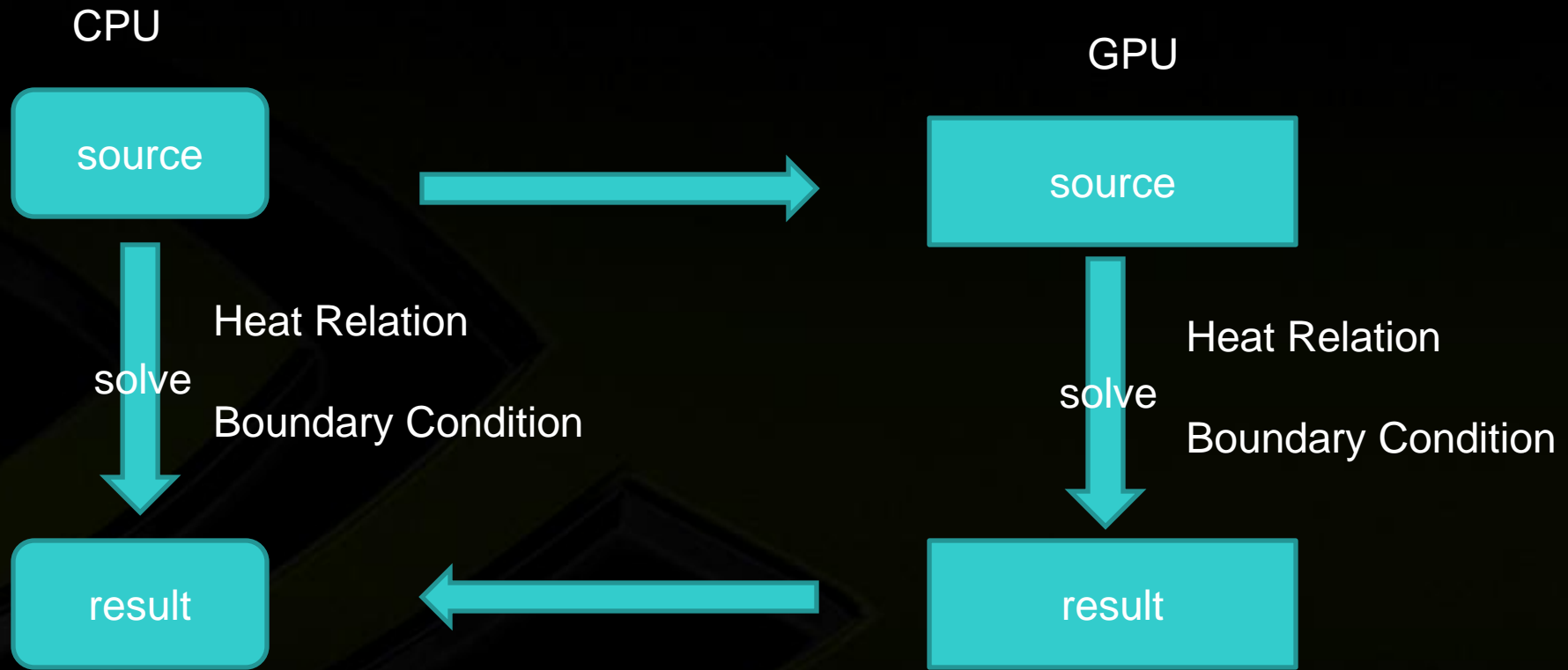


```
void __global__ functionG(double *u_dev, int meshsize, double r, double bound )
{
    int idx = blockIdx.x*blockDim.x+threadIdx.x;
    int i = idx+1;
    if(idx <meshsize +1 ) {

        u_dev[i]= i*0.01;
    }
    if(idx == 6){
    u_dev[idx+1]=u_dev[idx-1]=11;
    }
}

void compute_GPU( double * u_dev, double * u_host, double dt, double dx, double r, int mesh,
int print_step, int count, double time, double end_time, double bound )
{
    size_t gpuMemsize = sizeof(double)*(mesh+2);
    //for( int i=0; i < 6000 ; i++){ //time step
        functionG<<<4,5>>>(u_dev, mesh,r, bound);cudaErr2("kernel launch",1,0);
        cudaMemcpy(u_host, u_dev, gpuMemsize, cudaMemcpyDeviceToHost);cudaErr("memcpy u_dev to u_host");
        for(int i=0; i<mesh+1; i++){
            printf( " in kernel - GPU : temper[%d] ==> %f \n", i, u_host[i]);
        }
    //}
    return;
}
```

GPUcode-02 Solving



GPUcode-02 Malloc part



```
double* init_GPU_data(struct flow * temper, int mesh)
{
    double *u_dev; // for GPU data upload
    size_t gpuMemsize=sizeof(double)*(mesh+2) ;
    double *u_host; // temperal value
    cudaMalloc( (void**)&u_dev, gpuMemsize);cudaErr("malloc u_dev");
    u_host = (double *) malloc( gpuMemsize);
    for(int i=0;i<mesh;i++){
        u_host[i]= temper[i].old;
        printf("before %d : data initial :u_host[%d]= %f temper[%d].old  =%f\n", i, i, u_host[i], i,
temper[i].old);
    }
    cudaMemcpy(u_dev, u_host, gpuMemsize, cudaMemcpyHostToDevice);cudaErr("memcpy u_dev u_host");
    cudaMemcpy(u_host, u_dev, gpuMemsize, cudaMemcpyDeviceToHost);cudaErr("memcpy u_dev u_host");
    for(int i=0;i<mesh;i++){
        printf("after %d : data initial :u_host[%d]= %f temper[%d].old  =%f\n", i, i, u_host[i], i,
temper[i].old);
    }

    free(u_host);

    return (double *)u_dev;
}
```

GPUcode-02 Launch part



```
void __global__ functionG(double *u_dev, int meshsize, double r, double bound )
{
    int idx = blockIdx.x*blockDim.x+threadIdx.x;
    int i = idx+1;
    if(idx < meshsize +1 ) {
        u_dev[i] = (double) (1-2*r)*u_dev[i] + r*(u_dev[i-1] + u_dev[i+1] );
        // u_dev[i]= i*0.01;
    }
    if(idx == 6){
        u_dev[idx+1]=u_dev[idx-1]=11;
    }
}

void compute_GPU( double * u_dev, double * u_host, double dt, double dx, double r, int mesh,
                 int print_step, int count, double time, double end_time, double bound )
{
    size_t gpuMemsize = sizeof(double)*(mesh+2);
    //for( int i=0; i < 6000 ; i++){ //time step
        functionG<<<4,5>>>(u_dev, mesh,r, bound);cudaErr2("kernel launch",1,0);
        cudaMemcpy(u_host, u_dev, gpuMemsize, cudaMemcpyDeviceToHost);cudaErr("memcpy u_dev to u_host");
        for(int i=0; i<mesh+1; i++){
            printf( " in kernel - GPU : temper[%d] ==> %f \n", i, u_host[i]);
        }
    //}
    return;
}
```

How to Optimize?



Monte Carlo Simulation



Computation Time



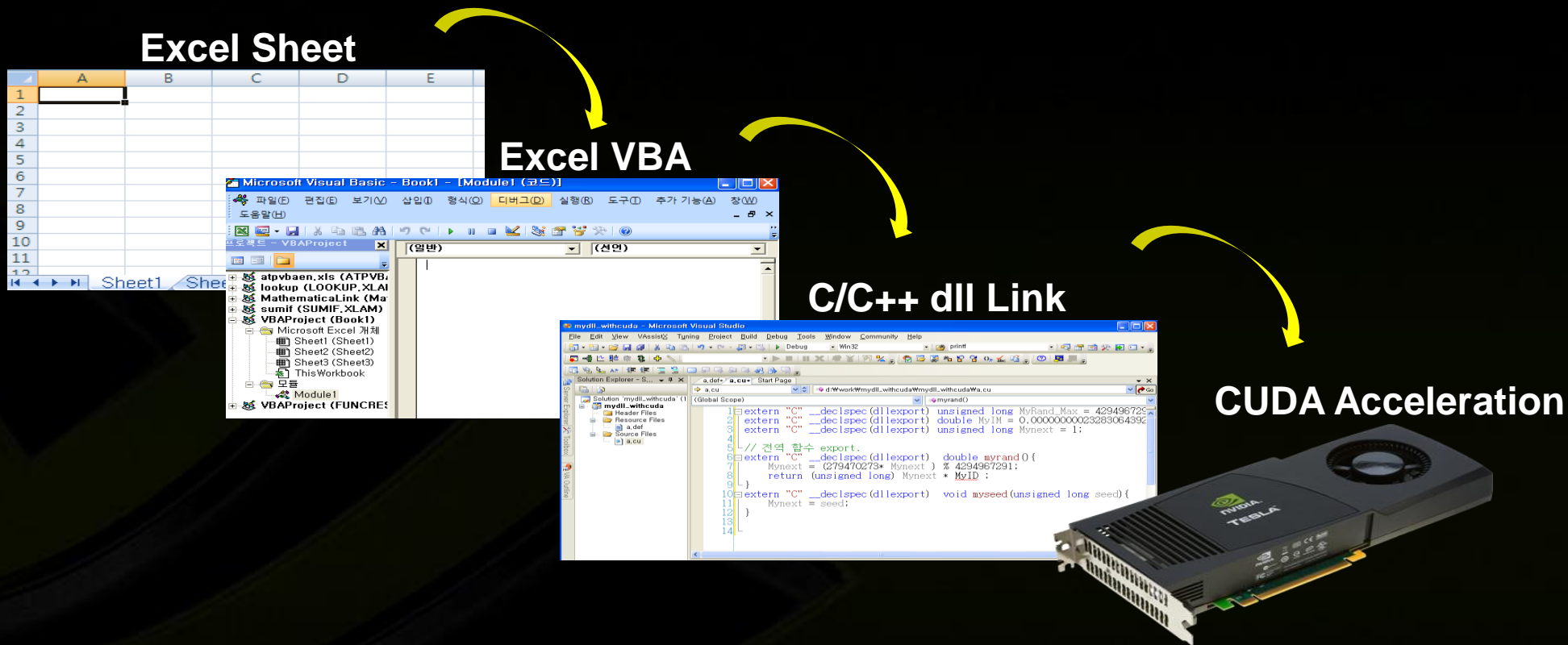
Malliavin MC results

type	Black-Sholes	5000	10000	50000	100000	200000	300000
Price	12.8216	12.8706	12.8721	12.8413	12.8525	12.8559	12.8480
Err	0.000%	0.38%	0.39%	0.15%	0.24%	0.27%	0.21%
Delta	0.5858	0.5724	0.5887	0.5826	0.5826	0.5829	0.5824
Err	0	2.28%	-0.51%	0.53%	0.53%	0.49%	0.58%
Gamma	0.0130	0.0112	0.0134	0.0127	0.0127	0.0127	0.0127
Err	0.00%	13.39%	-3.34%	2.07%	2.26%	2.21%	2.47%
Total Time	0:00	00:03	00:05	00:25	00:50	01:44	02:33



time : 100 sec
target1 : > 1 sec (100X)
Target2 : >0.001 sec (2000X)

Monte Carlo Simulation for Finance



Monte Carlo Code with VBA



```
function MC( S As Double, X As Double, T As Double, R As Double, _  
            Vol As Double, Q As Double, No As Double, rtype As String) As Double  
    Simul_No = No  
    dt = 1 / 365  
    For K = 0 To Simul_No - 1  
        Juga = S  
        For i = 0 To MaxStep - 1  
            Juga = Juga * Exp( (R - Q - Vol ^ 2 / 2) * dt + Vol * Sqr(dt) * MakeNorsD() )  
        Next  
        price = Exp(-R * T) * Max(Juga - X, 0)  
        sum_price = sum_price + price  
    Next  
    MC = sum_price / Simul_No  
End function
```

Malliavin Greeks



- **Greek computation for Monte Carlo simulation**

$$\frac{\partial}{\partial s} E[f(S)] \approx \frac{E[f(S_0 + \Delta S)] - E[f(S_0)]}{\Delta S}$$

- **Malliavin approach**

$$\frac{\partial}{\partial s} E[f(S)] \approx E[f(S) \cdot \underline{W(S)}]$$

Malliavin weights

With Malliavin approach, we can save the computation time.

Problem



To compare the accuracy, we compute the Price, Delta and Gamma of Vanilla Call option.

Approach

1. Closed Form solution (VBA,C)
2. Monte (VBA, C)
3. Malliavin (VBA,C, **CUDA v1, v2**)

Malliavin Monte Carlo Code with VBA



```
function Malliavin( S As Double, X As Double, T As Double, R As Double, _  
                  Vol As Double, Q As Double, No As Double, rtype As String) As Double
```

```
    Simul_No = No
```

```
    dt = 1 / 365
```

```
    For K = 0 To Simul_No - 1
```

```
        Juga = S
```

```
        For i = 0 To MaxStep - 1
```

```
            Juga = Juga * Exp( (R - Q - Vol ^ 2 / 2) * dt + Vol * Sqr(dt) * MakeNorsD() )
```

```
        Next
```

```
        WT = (Log(Juga) - Log(S) - (R - Q - 1 / 2 * Vol ^ 2) * T) / Vol
```

```
        WT_delta = (WT / (S * Vol * T))
```

```
        WT_gamma = (1 / (Vol * T * S ^ 2)) * (WT ^ 2 / (Vol * T) - WT - 1 / Vol)
```

```
        price = Exp(-R * T) * Max(Juga - X, 0)
```

```
        delta = Exp(-R * T) * Max(Juga - X, 0) * WT_delta
```

```
        gamma = Exp(-R * T) * Max(Juga - X, 0) * WT_gamma
```

```
    sum_price = sum_price + price
```

```
    sum_delta = sum_delta + delta
```

```
    sum_gamma = sum_gamma + Gamma
```

```
    Next
```

```
    Malliavin = sum_delta / Simul_No
```

```
    NVIDIA Confidential
```

```
End function
```

Step1 Malliavin Monte Carlo C language sketch



```
void Malliavin( double S , double X , double T , double R, double Vol, double Q, long No){
    long Simul_No = No;
    double dt = 1; // dt = 1 / 365
    for ( int K = 0; K < Simul_No - 1 ; K++){
        Juga = S ;
        for (int i = 0; i < MaxStep - 1 ; i++){
            Juga = Juga * exp( (R - Q - Vol ^ 2 / 2) * dt + Vol * sqrt(dt) * norm0 ); // rand with box muller
        }

        WT = (Log(Juga) - Log(S) - (R - Q - 1 / 2 * Vol ^ 2) * T) / Vol;
        WT_delta = (WT / (S * Vol * T));
        WT_gamma = (1 / (Vol * T * S ^ 2)) * (WT ^ 2 / (Vol * T) - WT - 1 / Vol);

        price = Exp(-R * T) * max(Juga - X, 0);
        delta = Exp(-R * T) * max(Juga - X, 0) * WT_delta;
        gamma = Exp(-R * T) * max(Juga - X, 0) * WT_gamma;
        sum.price = sum.price + price;
        sum.delta = sum.delta + delta;
        sum.gamma = sum.gamma + Gamma ;
    }
    r.price = sum.price / Simul_No
    r.delta = sum.delta / Simul_No
    r.gamma = sum.gamma / Simul_No
}
```

Step2 Sketch (kernel part)

```

Void __global__ Malliavin_compute( double S , double X , double T , double R, double Vol, double Q, long No){
    long Simul_No = No;
    double dt = 1; // dt = 1 / 365
    for ( int K = 0; K < Simul_No - 1 ; K++){
        Juga = S ;
        for (int i = 0; i < MaxStep - 1 ; i++){
            Juga = Juga * exp( (R - Q - Vol * Vol / 2) * dt + Vol * sqrt(dt) * norm(k,i) );
        }

        WT = (log(Juga) - log(S) - (R - Q - 1 / 2 * Vol * Vol) * T) / Vol;
        WT_delta = (WT / (S * Vol * T));
        WT_gamma = (1 / (Vol * T * S * S)) * (WT * WT / (Vol * T) - WT - 1 / Vol);

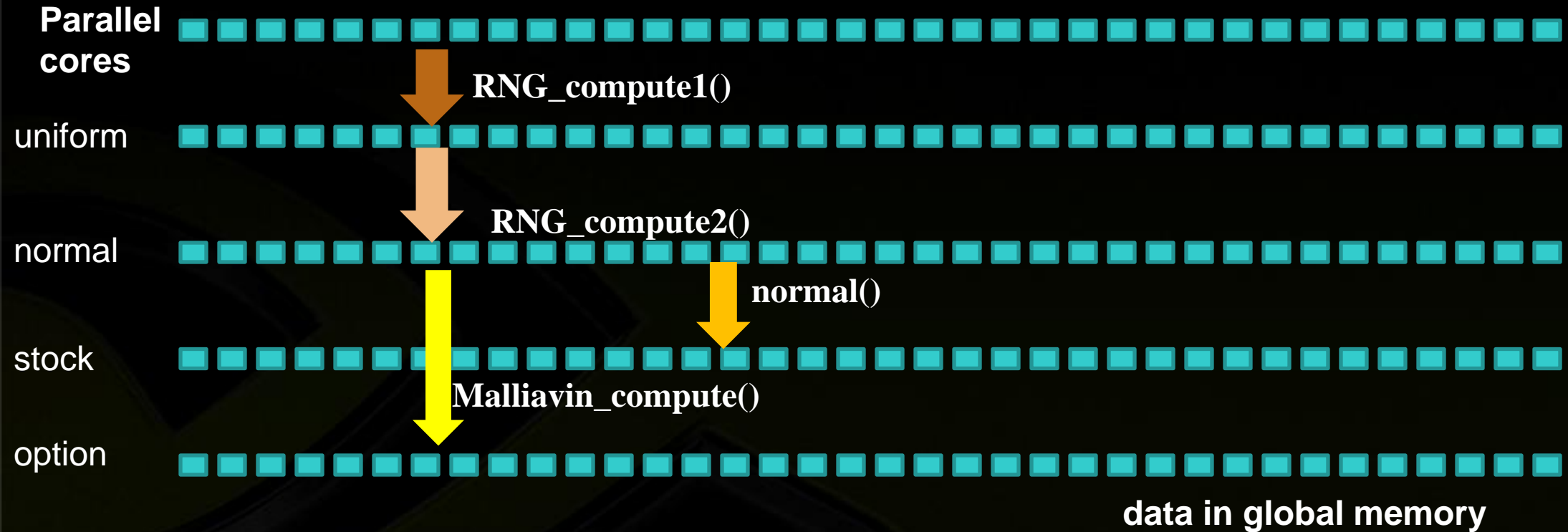
        price = Exp(-R * T) * max(Juga - X, 0);
        delta = Exp(-R * T) * max(Juga - X, 0) * WT_delta;
        gamma = Exp(-R * T) * max(Juga - X, 0) * WT_gamma;
        sum.price = sum.price + price;
        sum.delta = sum.delta + delta;
        sum.gamma = sum.gamma + Gamma ;
    }
    r.price = sum.price / Simul_No
    r.delta = sum.delta / Simul_No
    r.gamma = sum.gamma / Simul_No
}

```

Simm_No =
Total Sim / (N threads * M blocks)

Real Price =
Sum (r.price) / (N * M)

Step2 Parallel Memory Map



```
(float *) __device__ normal(int k, int, j, int size_j, float * normal) {
```

```
    int index = k*size_j + j;
```

```
    return &normal[index];
```

Step2 Sketch (host part)



```
#include <stdio.h>

__global__ RNG_compute(parameter);
__global__ Malliavin_compute(parameter);

main(){

    malloc(); //cpu malloc
    cudaMalloc(); //GPU malloc
    cudaMemcpy(); // transfer

    RNG_compute 1<<<N,M>>> (parameter); // generate RNG (uniform)

    RNG_compute2 <<<N,M>>> (parameter); // generate RNG ( BM,Moro)

    Malliavin_compute <<<N,M>>> (parameter); // simulation

    cudaMemcpy(); //get results

    return 0;

}
```


Step2 Malliavin Monte Carlo CUDA language sketch (rng part1)



```
__global__
static void RNG_rand48_get_int(uint2 *state, int *res, int num_blocks, uint2 A, uint2 C)
{
    const int nThreads = blockDim.x * gridDim.x;

    int  nOutIdx = threadIdx.x + blockIdx.x * blockDim.x;
    uint2 lstate = state[nOutIdx];
    int i;
    for (i = 0; i < num_blocks; ++i) {

        res[nOutIdx] = ( lstate.x >> 17 ) | ( lstate.y << 7);
        nOutIdx += nThreads;

        lstate = RNG_rand48_iterate_single(lstate, A, C);
    }

    nOutIdx = threadIdx.x + blockIdx.x * blockDim.x;
    state[nOutIdx] = lstate;
}
```

Step2 Malliavin Monte Carlo CUDA language sketch (rng part2)



```
Void __global__ RNG_compute(int * uniform , float * normal, int length){

int index = blockDim.x*blockIdx.x+threadIdx.x;

__shared__ int s[i];
__shared__ float s_r[i];

if( threadIdx.x ==0){
    for (int i = 0 ; i<blockDim.x ; i++){
        s[i]=uniform[blockDim.x*blockIdx.x + i]; // load uniform
    }
}
s_r[threadIdx.x] = (float) moro(s[threadIdx.x]); // moro inversion with parallel

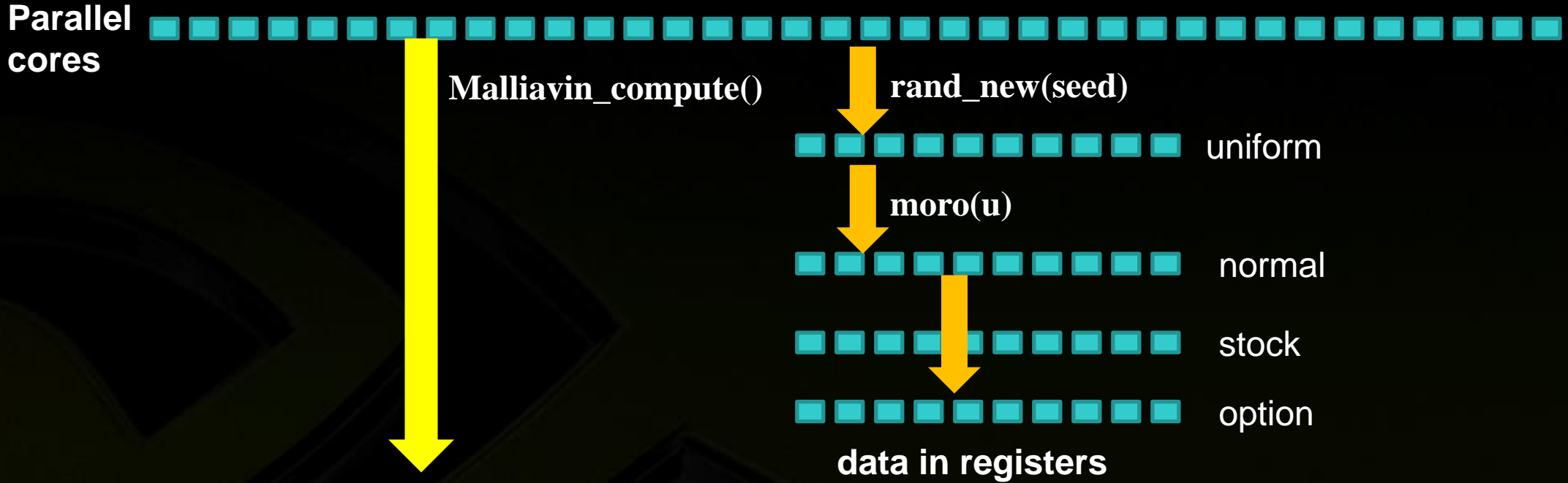
if( threadIdx.x ==0){
    for (int i = 0 ; i<blockDim.x ; i++){
        s[blockDim.x*blockIdx.x+i]= s_r[i] ]; // save normal
    }
}

}
```

Box-Muller vs Moro Inversion

```
__device__ void BoxMuller(float& u1, float& u2){
    float r = sqrtf(-2.0f * logf(u1));
    float phi = 2 * PI * u2;
    u1 = r * __cosf(phi);
    u2 = r * __sinf(phi);
}
__device__ Moro( float u ){
    // skip the const value
    x = u - 0.5;
    if (abs(x) < 0.42) {
        r = x * x;
        r = x * (((a4 * r + a3) * r + a2) * r + a1) / (((b4 * r + b3) * r + b2) * r + b1) * r + 1);
    } else{
        if (x > 0) r = log(-log(1 - u));
        if (x <= 0) r = log(-log(u));
        r = c1 + r * (c2 + r * (c3 + r * (c4 + r * (c5 + r * (c6 + r * (c7 + r * (c8 + r * c9))))));
        if (x <= 0) r = -r;
    }f
    return r;
}
```

Step3 New approach for Direct LCG and Parallel Memory Map



```
double rand_new( int next){  
    next = (a * next + b) % M;  
    return (double) next * (1/M);  
}
```

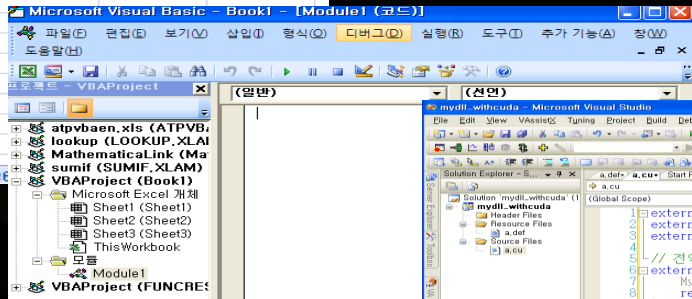
Platform for finance



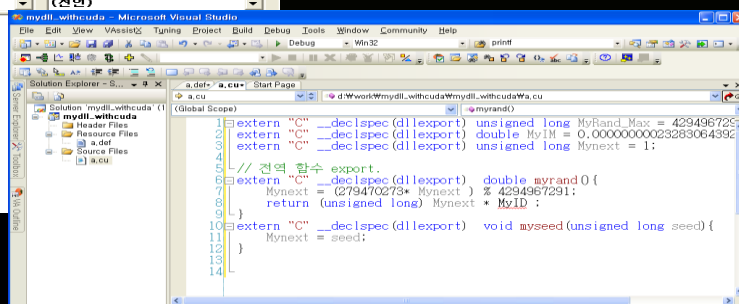
Excel Sheet

	A	B	C	D	E
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					
12					

Excel VBA



C/C++ dll Link



Socket communication

Grid Manager

12*8 GPU*448 core
= 43,000 core



GPU cluster

12 nodes system (1/2 for backup)

8 GPU per node

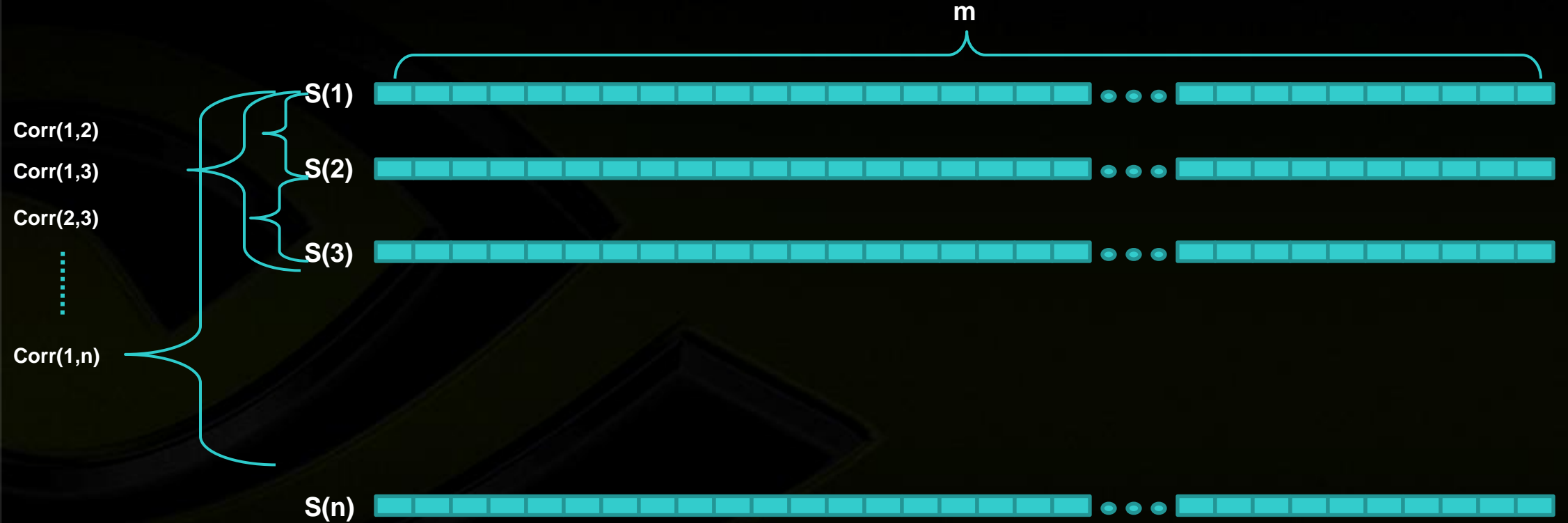
How to Optimize?



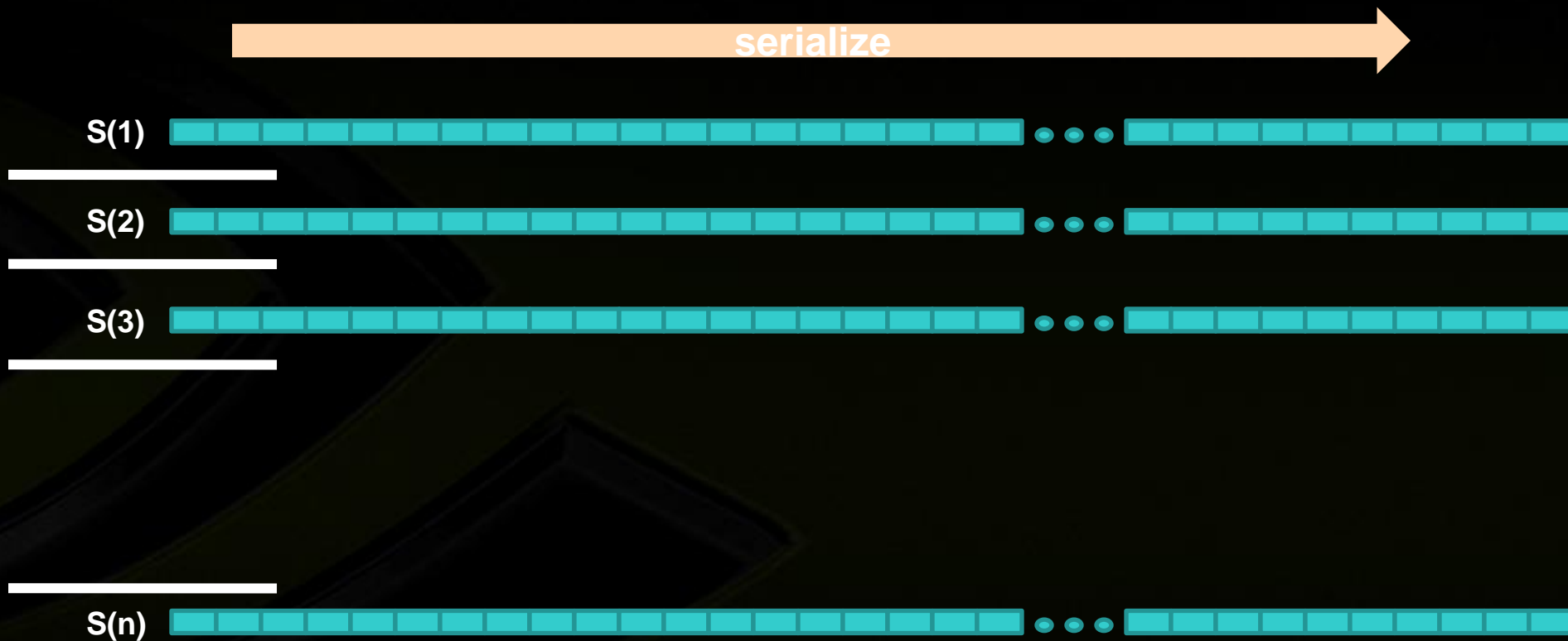
Time Series Analysis



Multivariate Time-series

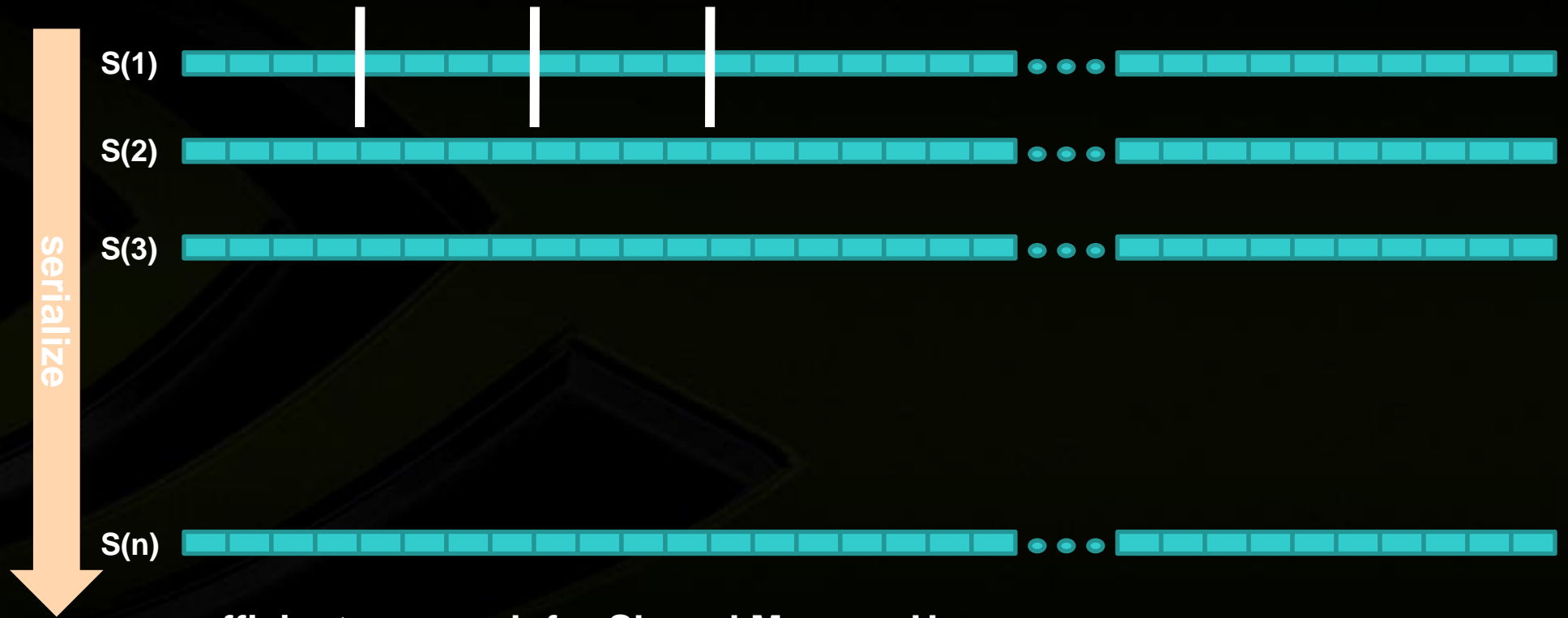


How to parallelize with CUDA



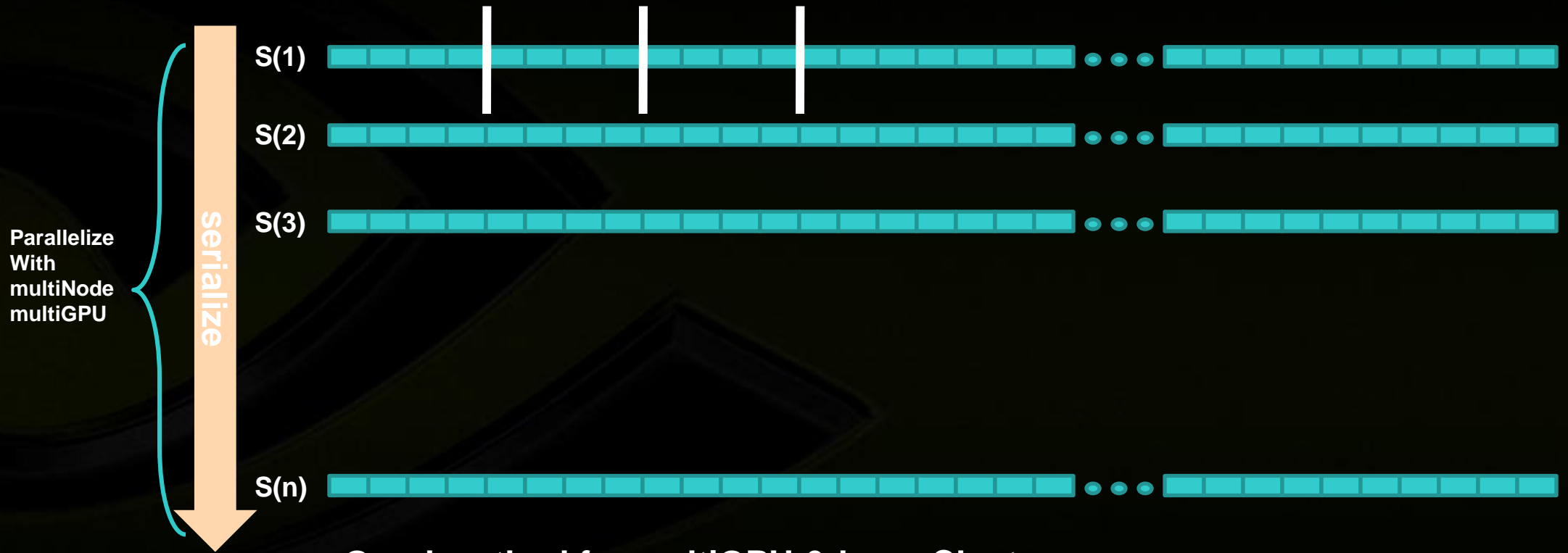
Unefficient approach for Shared Memory Usage

How to parallelize with CUDA



efficient approach for Shared Memory Usage
Need to reduction technique for mean etc.

How to parallelize with CUDA



Good method for multiGPU & large Cluster

In pair(I,J), Pearson Correlation Coefficient



$$\rho = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2 \sum (y_i - \bar{y})^2}}$$

$$\sum (x_i - \bar{x})(y_i - \bar{y}) = \sum x_i y_i - \bar{x} \sum y_i - \bar{y} \sum x_i + \sum \bar{x} \bar{y}$$

$$= \sum x_i y_i - n \bar{x} \bar{y}$$

$$\sum (x_i - \bar{x})^2 = \sum x_i^2 - n \bar{x}^2$$

$$\sum (y_i - \bar{y})^2 = \sum y_i^2 - n \bar{y}^2$$

$$\rho = \frac{\sum x_i y_i - n \bar{x} \bar{y}}{\sqrt{(\sum x_i^2 - n \bar{x}^2)(\sum y_i^2 - n \bar{y}^2)}}$$

**We can parallelize the Sumation !!
After summation, find the mean.**

How to parallelize with CUDA : Flow Chart



Method 1

Input A, B

- ↓ Find the mean of A, B
start to **sum** (Ai), (Bi)
- ↓ Find Cov(A,B), Cov(A,A), Cov(B,B)
start to **sum** (Ai,Bi), (Ai^2), (Bi^2) with mean

Benefit : Easy to implementation
with two function

R+CUDA project

<http://brainarray.mbni.med.umich.edu/Brainarray/rgpgpu/>

Method 2

Input A, B

Start to **sum** (Ai), (Bi), (Ai,Bi), (Ai^2), (Bi^2)

Find the mean of A, B

Find Cov(A,B), Cov(A,A), Cov(B,B)

Benefit : oneshot sum (speed-up)

In pair(i,j), Pearson Correlation Coefficient

FOR (i, j) – pair : serial

**FOR k (time-series) : parallel
compute**

reduction for results

compute mean(i), mean(j),

compute cov(i,j), cov(i,i) ,cov(j,j)

compute corr(i,j)



In pair(i,j), Pearson Correlation Coefficient

FOR (i, j) – pair : serial

FOR k (time-series) : parallel

FOR shared memory

compute

reduction for results

compute mean(i), mean(j),

compute cov(i,j), cov(i,i) ,cov(j,j)

compute corr(i,j)

How to Optimize?



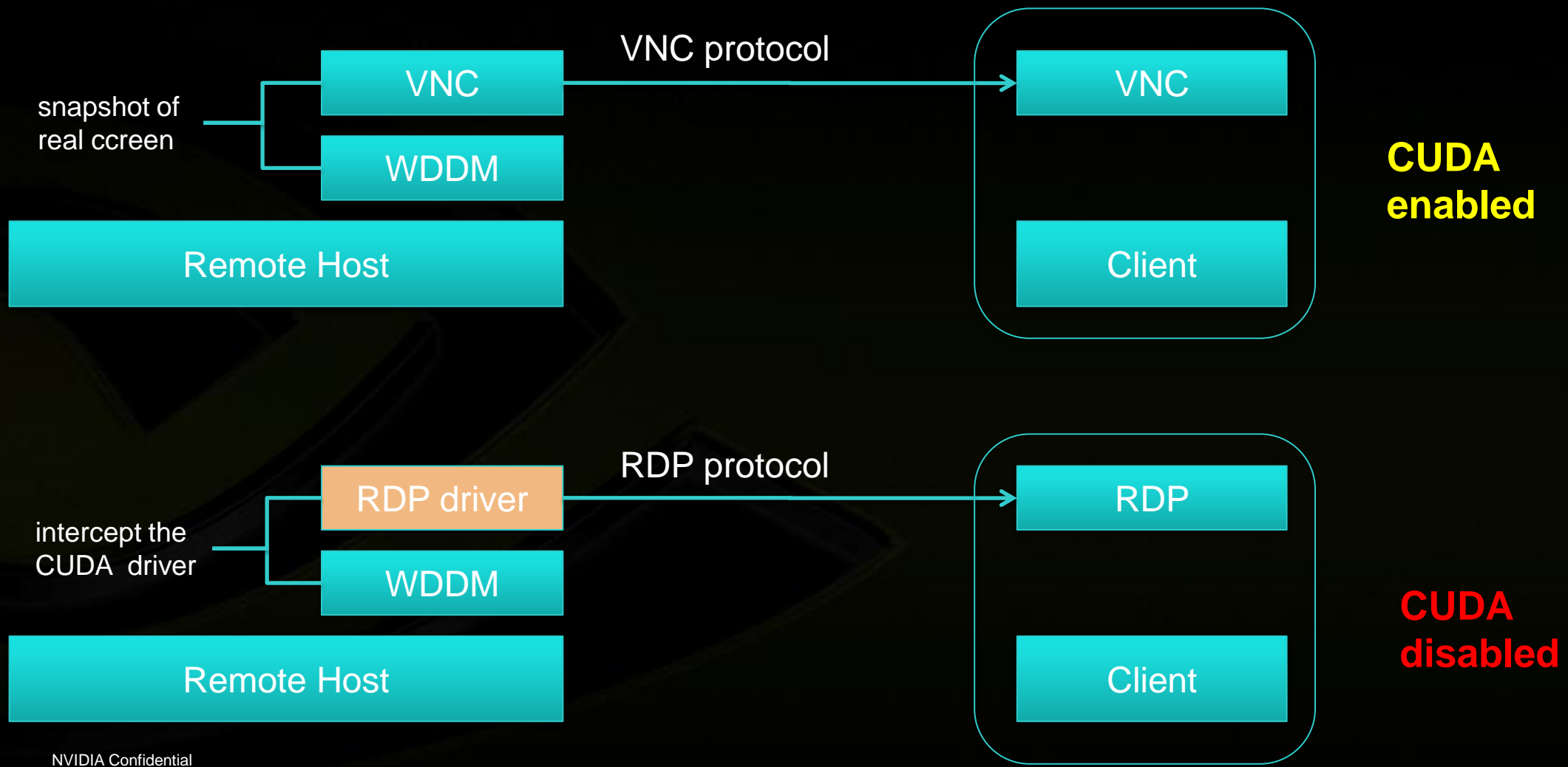
Focus on Optimization



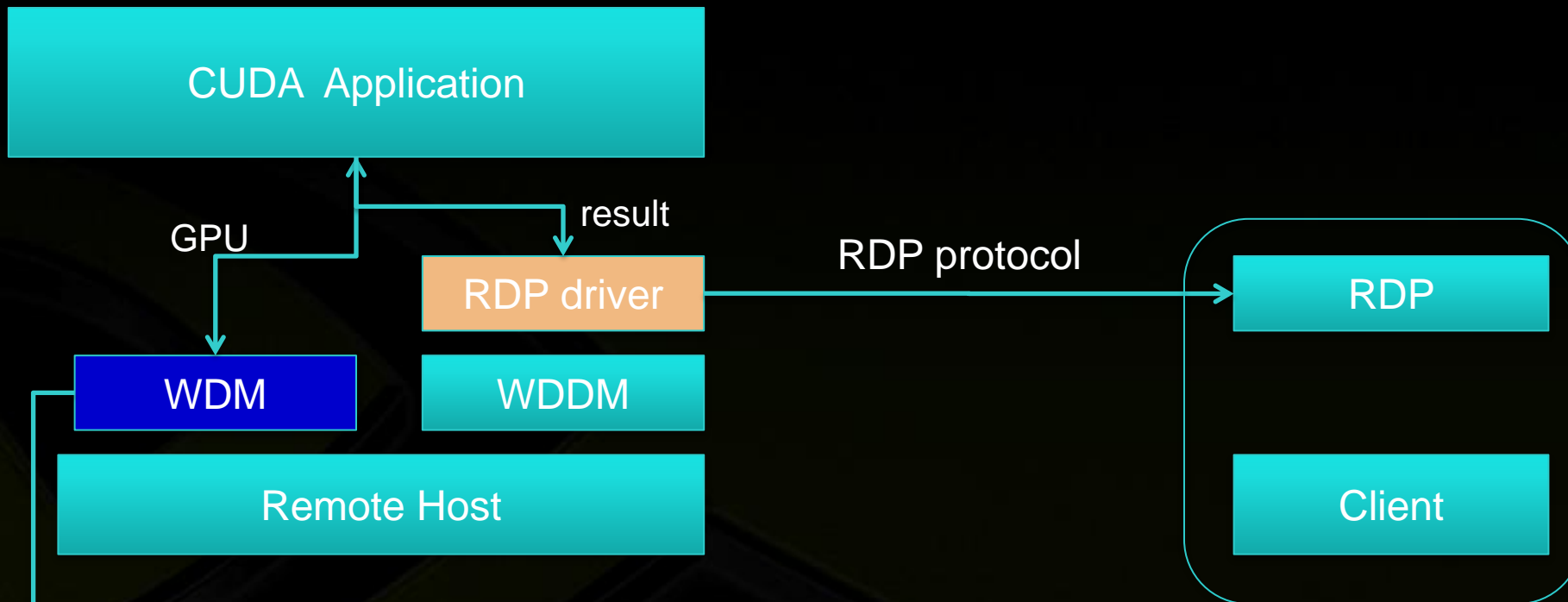
System Optimization Tips before start CUDA programming



Remote Use



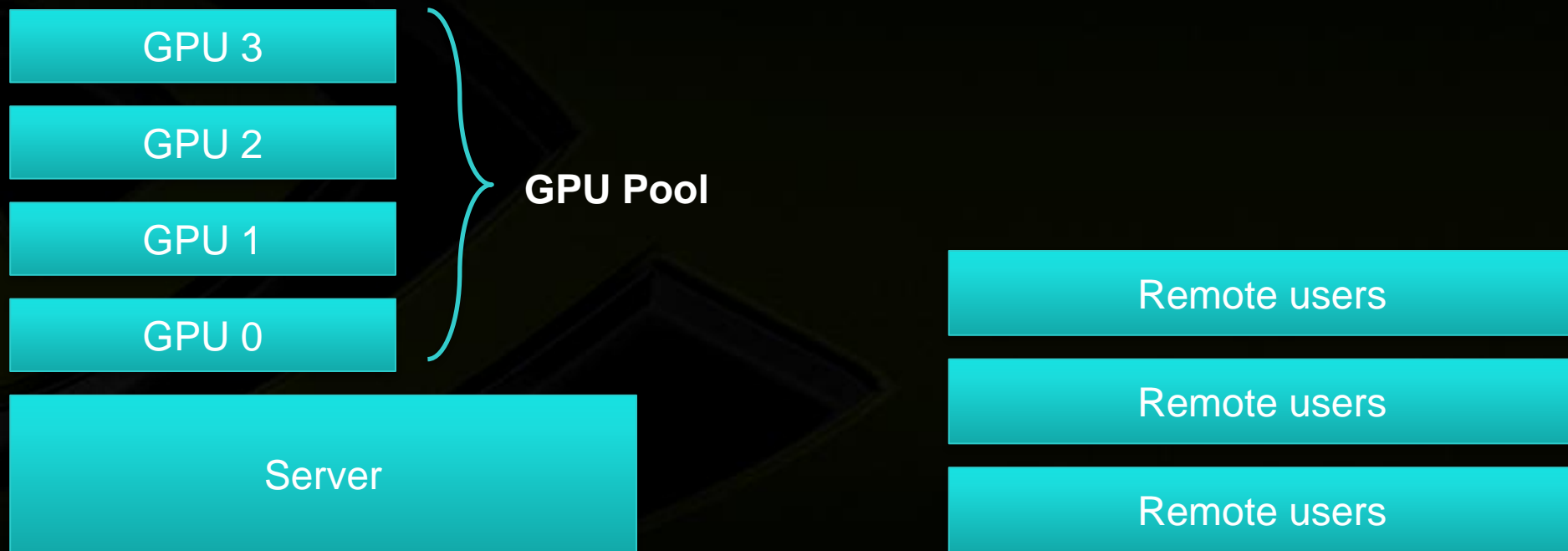
TCC driver



CUDA enabled

- Remote Desktop
- Kernel Launch Over Head
- GPU exclusive Mode
- Windows Service for Session0/1
- large single Malloc

GPU exclusive Mode for multiuser

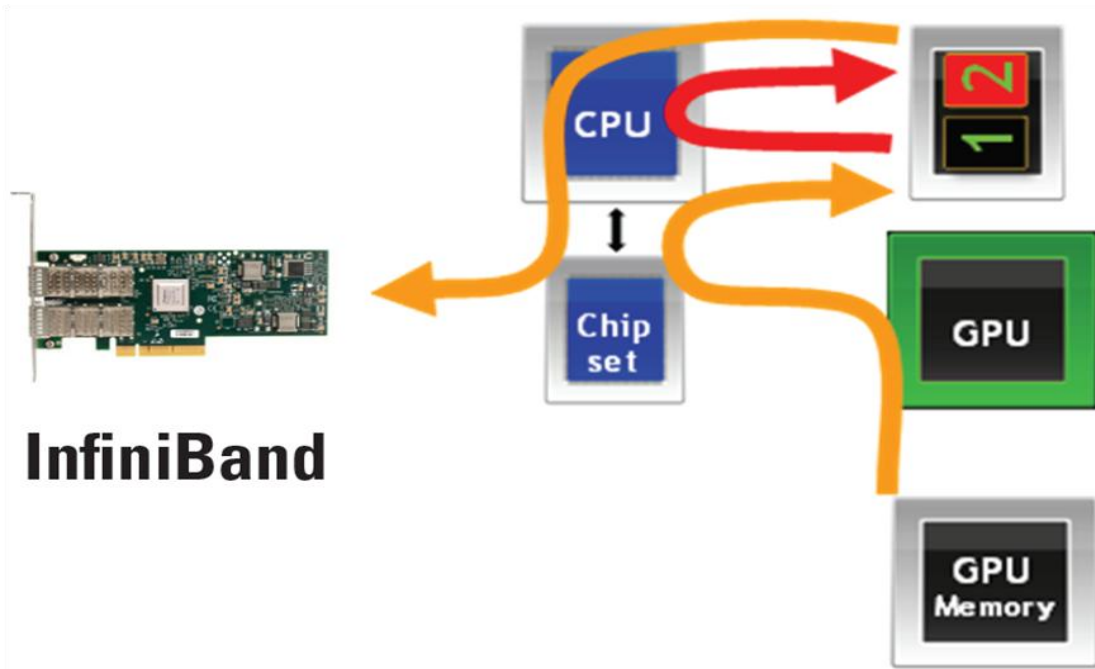


GPUDirect for GPU cluster

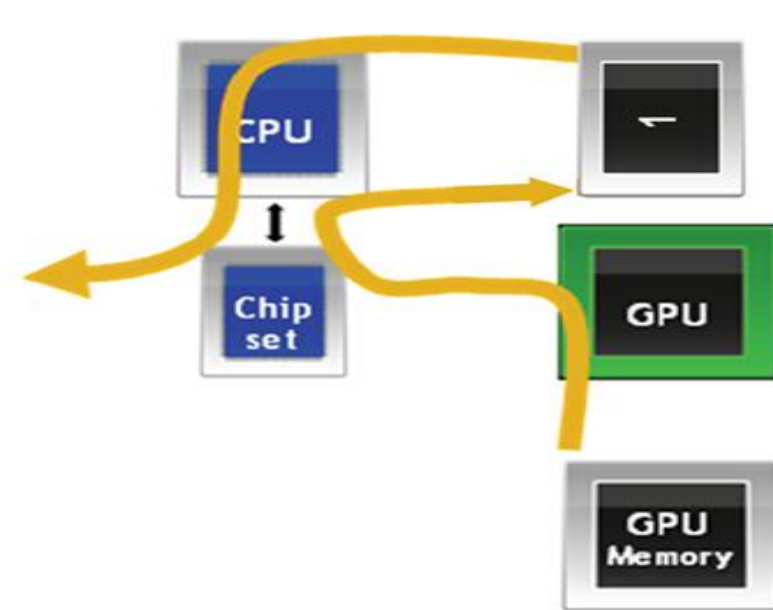


MPI Communication

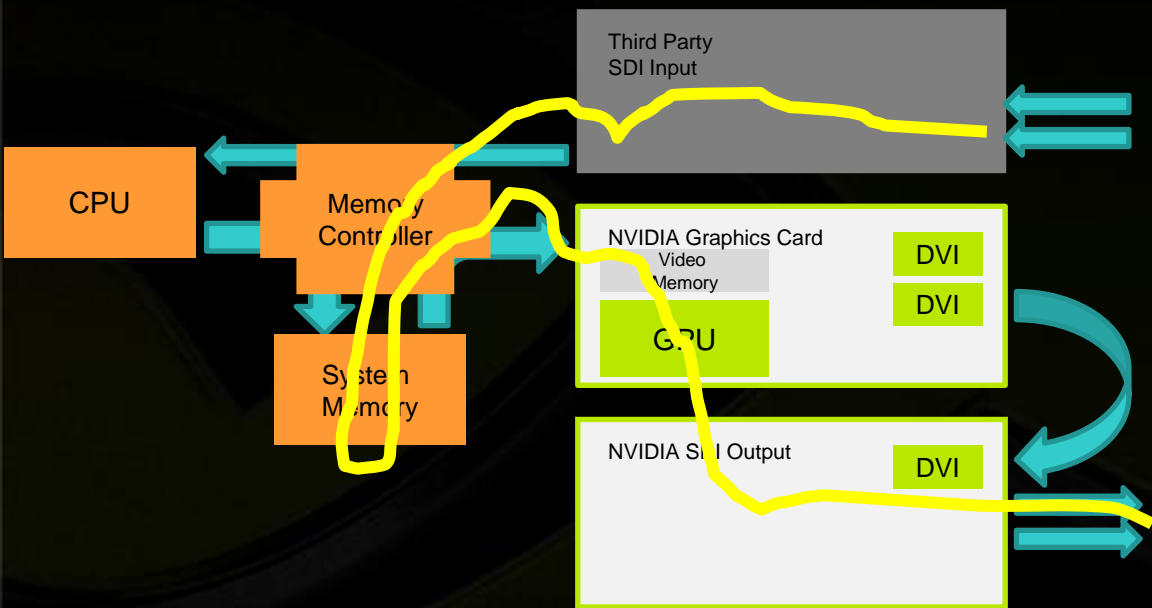
System Pinned Memory
System Pageable Memory



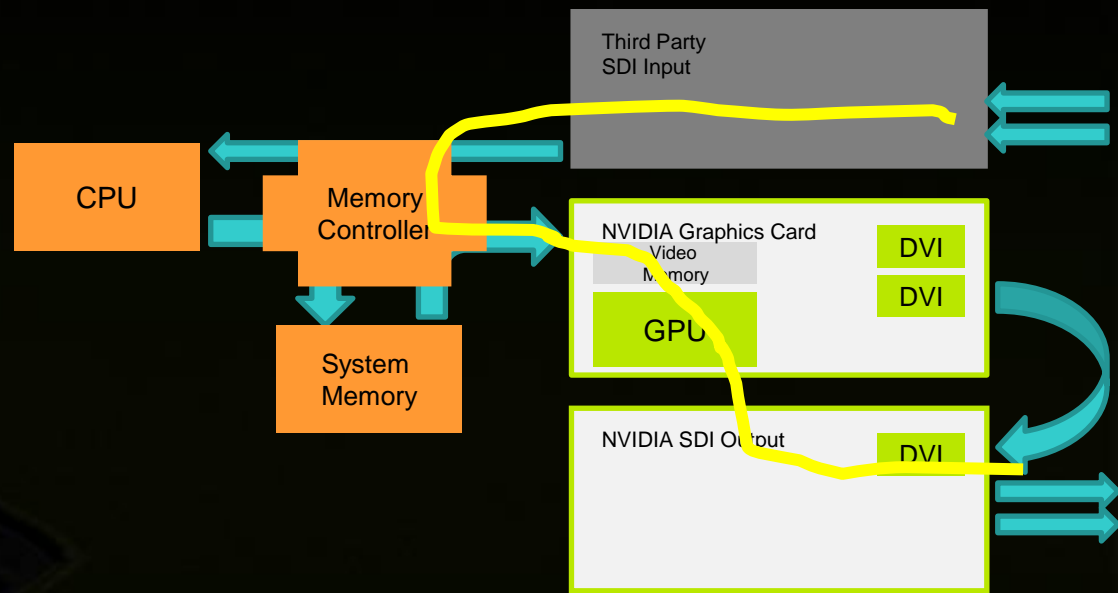
System Pinned Memory share



FBO on SDI with Quadro



Write to Host memory and to write GPU memory



Direct write to OpenGL Frame Buffer Object

Conceptual Tips for CUDA optimization



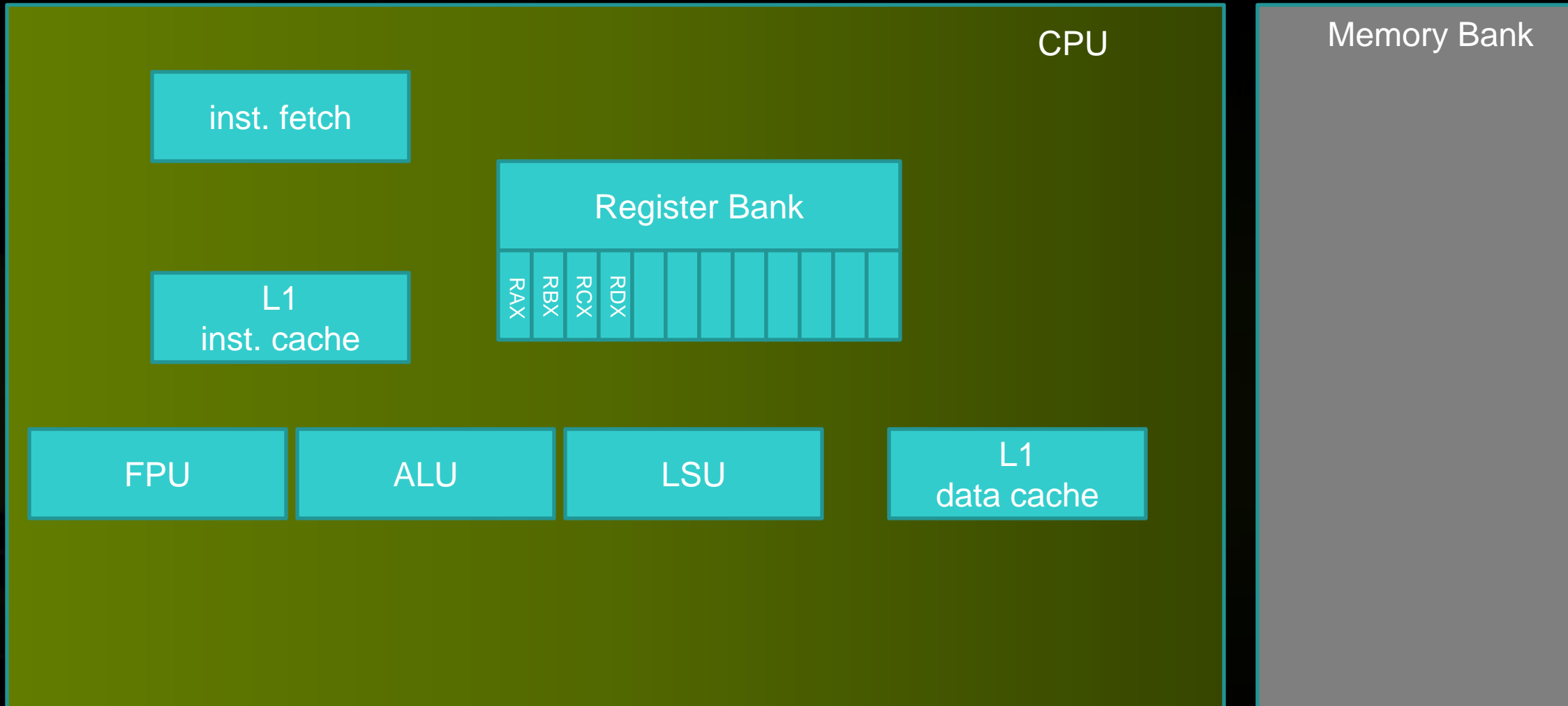


SIMT architecture

Single Instruction Multiple Threads

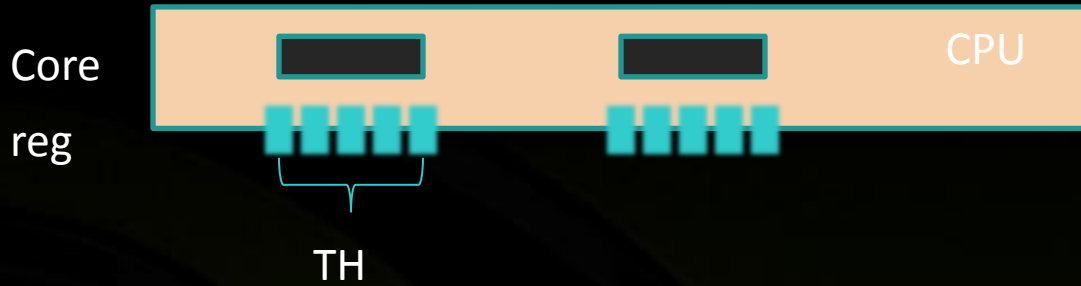


Abstract overview of CPU Core

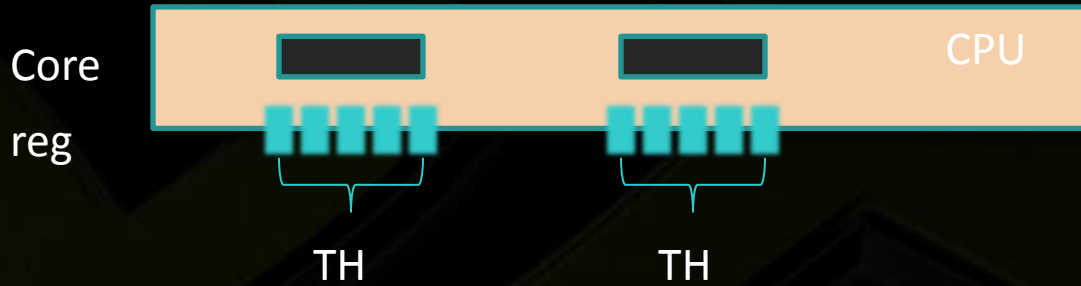


Threads on Multicore CPU

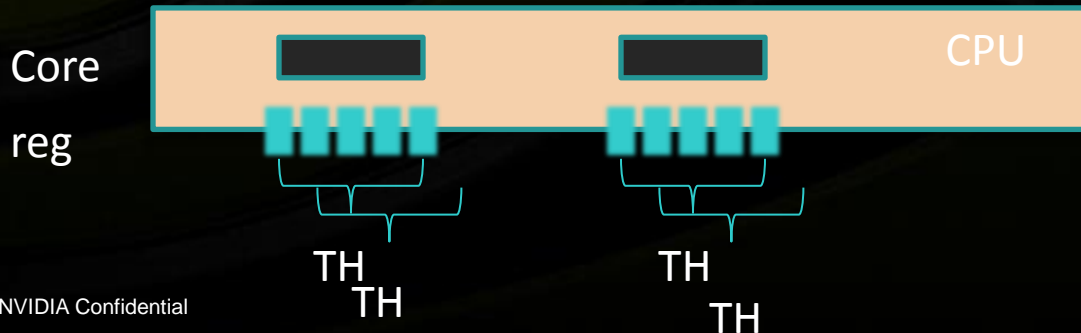
CPU



General Programming



Winthread, pthread

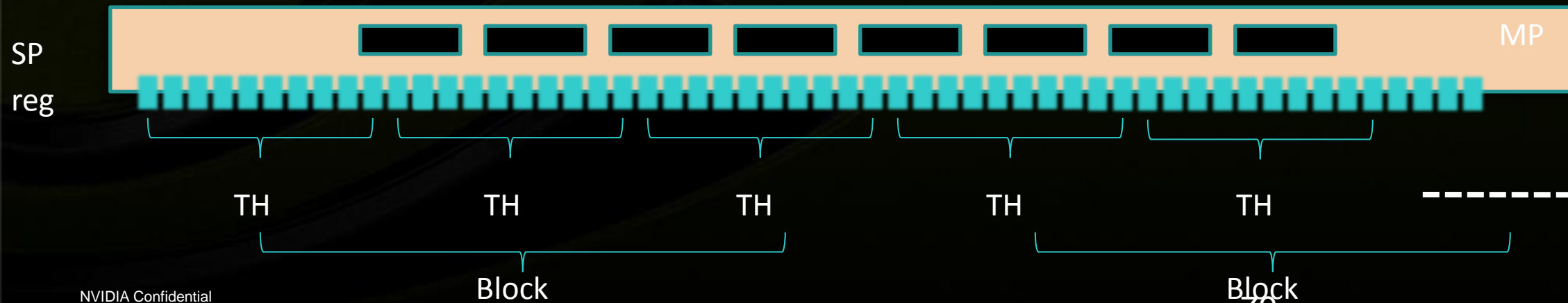
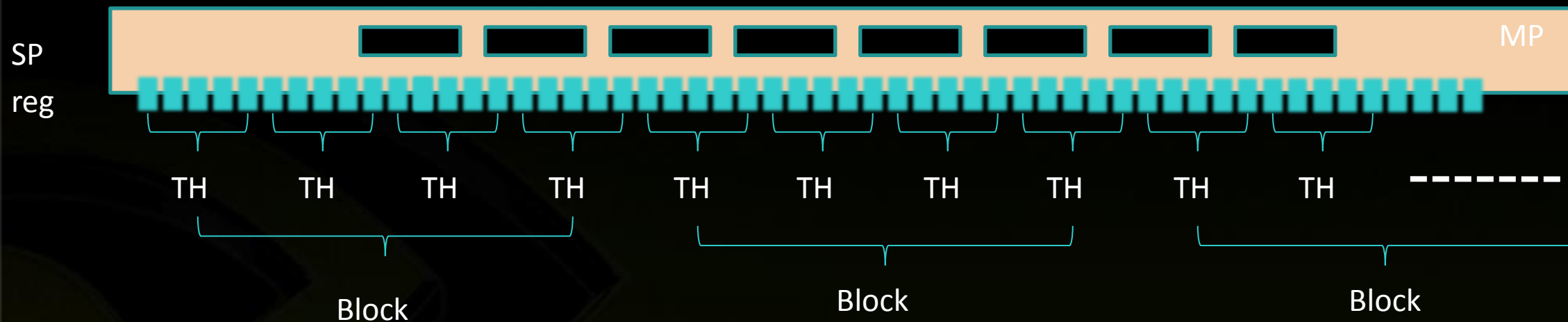


Hyper-threading

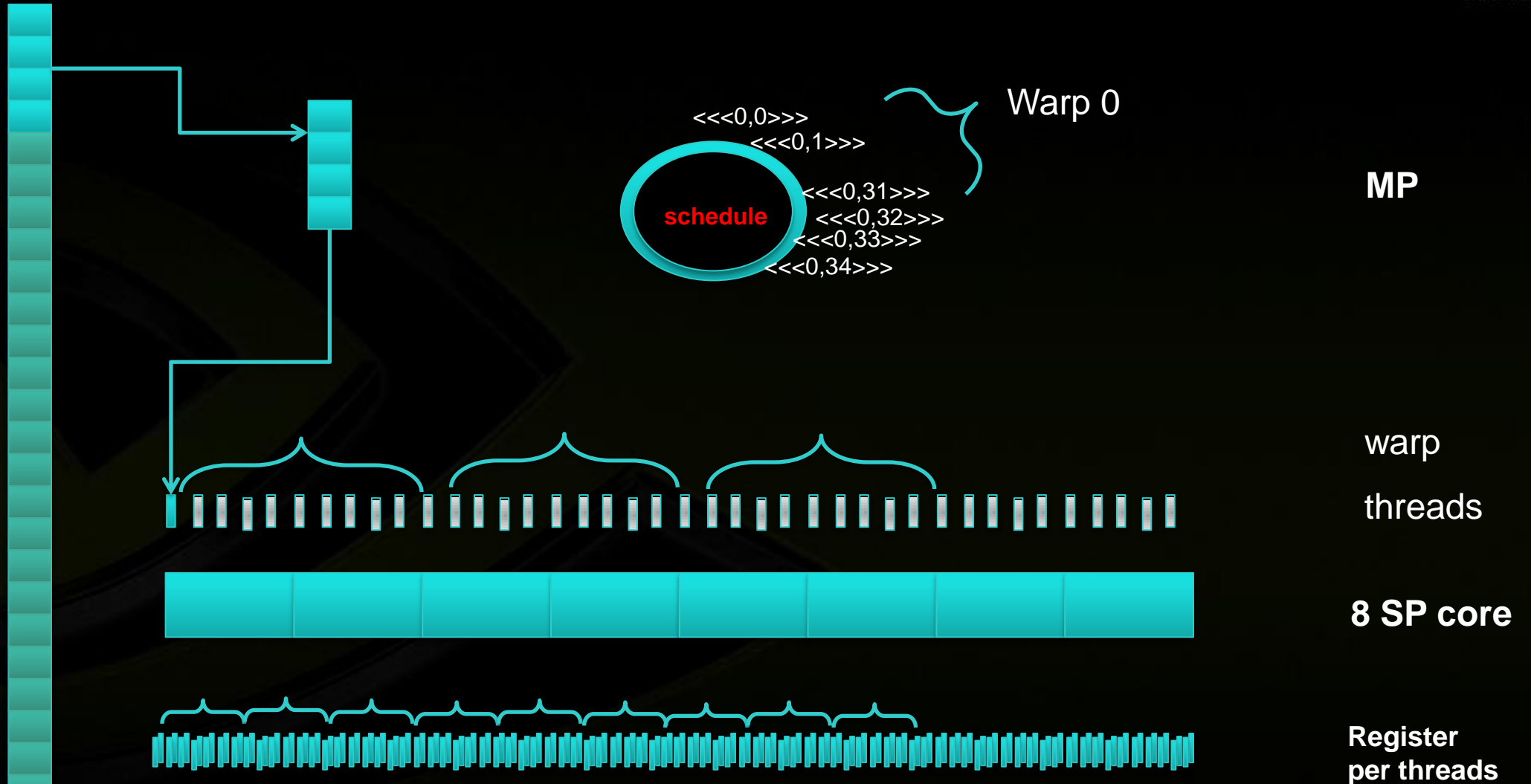
Threads on Manicore GPU



H/W Multi Processor vs S/W Active Block



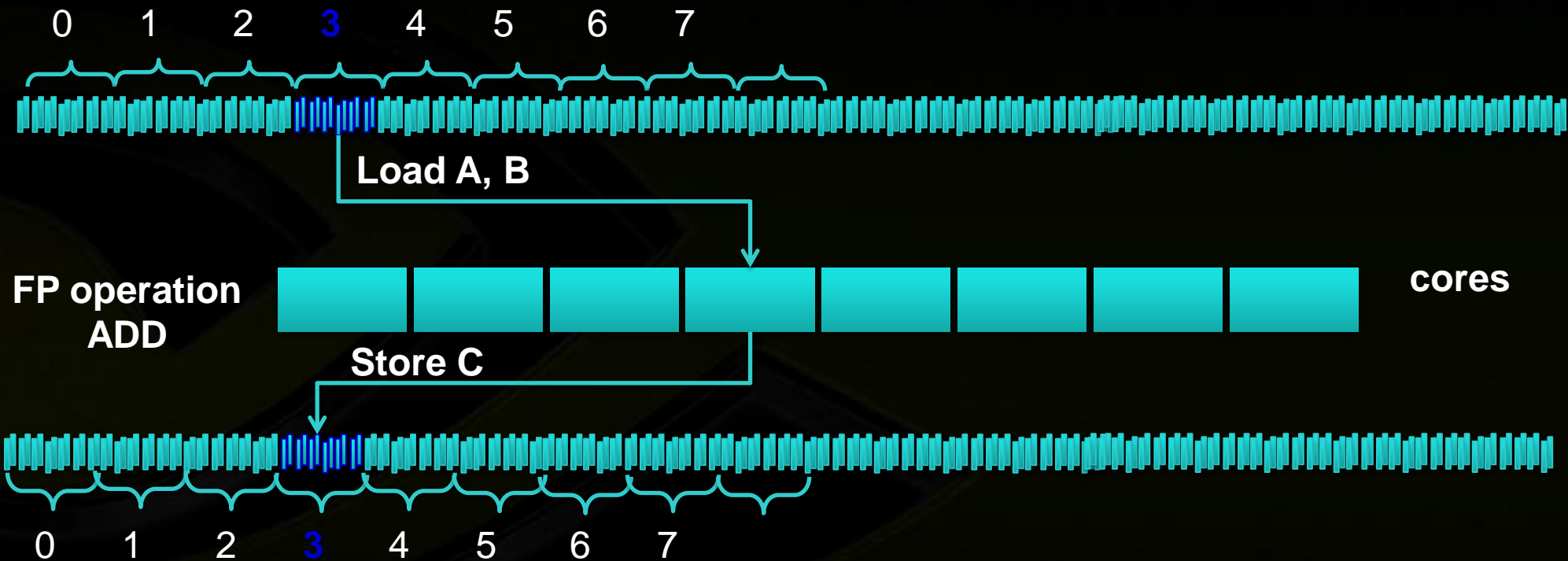
Overview of WARP schedule



Overview of Instruction Fetch



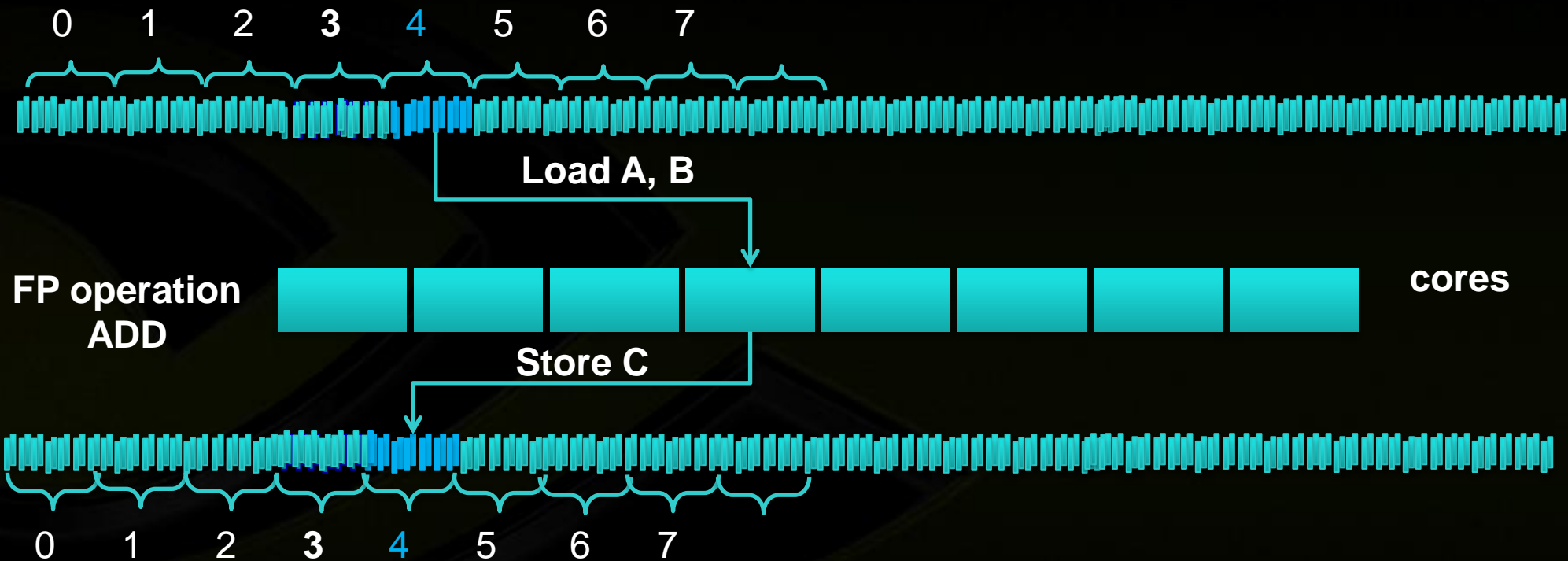
blockIdx.x=0, threadIdx.x=3



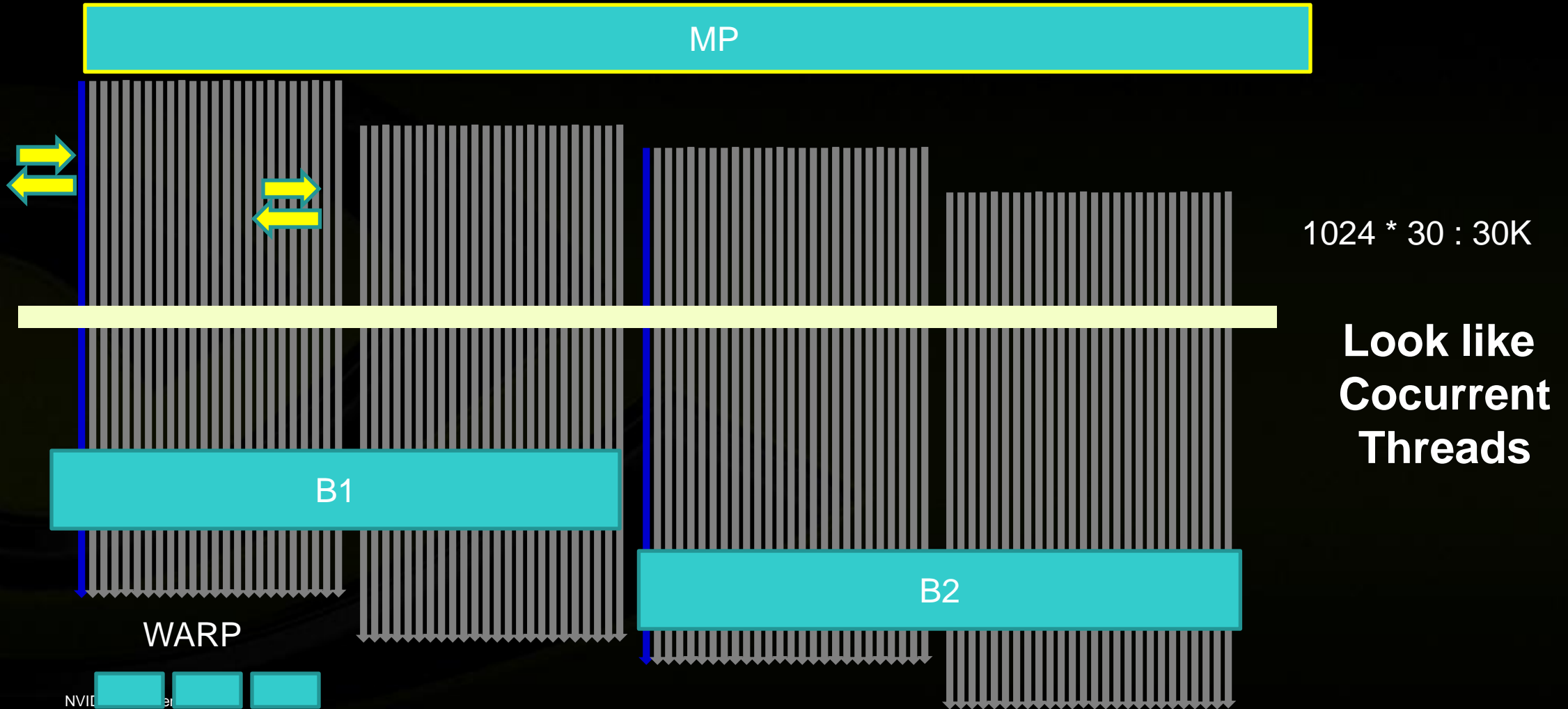
Overview of Instruction Fetch



blockIdx.x=0, threadIdx.x=4



Thread schedule within MP : WARP



Occupancy Calculator on CUDA SDK



CUDA GPU Occupancy Calculator

[Click Here for detailed instructions on how to use this occupancy calculator.](#)
[For more information on NVIDIA CUDA, visit http://developer.nvidia.com/cuda](http://developer.nvidia.com/cuda)

Just follow steps 1, 2, and 3 below! (or click here for help)

1.) Select Compute Capability (click): 2.0 [\(Help\)](#)

2.) Enter your resource usage:

Threads Per Block	256	(Help)
Registers Per Thread	8	
Shared Memory Per Block (bytes)	1024	

(Don't edit anything below this line)

3.) GPU Occupancy Data is displayed here and in the graphs:

Active Threads per Multiprocessor	1536	(Help)
Active Warps per Multiprocessor	48	
Active Thread Blocks per Multiprocessor	6	
Occupancy of each Multiprocessor	100%	

Physical Limits for GPU Compute Capability: 2.0

Threads per Warp	32
Warps per Multiprocessor	48
Threads per Multiprocessor	1536
Thread Blocks per Multiprocessor	8
Total # of 32-bit registers per Multiprocessor	32768
Register allocation unit size	64
Register allocation granularity	warp
Shared Memory per Multiprocessor (bytes)	49152

Your chosen resource usage is indicated by the red triangle on the graphs. The other data points represent the range of possible block sizes, register counts, and shared mem

Varying Block Size

My Block Size 256

Threads Per Block	Multiprocessor Warp Occupancy
16	8
32	16
64	24
128	32
256	48
512	48

Varying Register Count

My Register Count 8

Register Count	Multiprocessor Warp Occupancy
0	16
8	48
16	48
24	32
32	24
40	16

CUDA profiler on CUDA toolkit



test2 - Compute Visual Profiler - [Session1 - Device_0 - Context_0 [CUDA]]

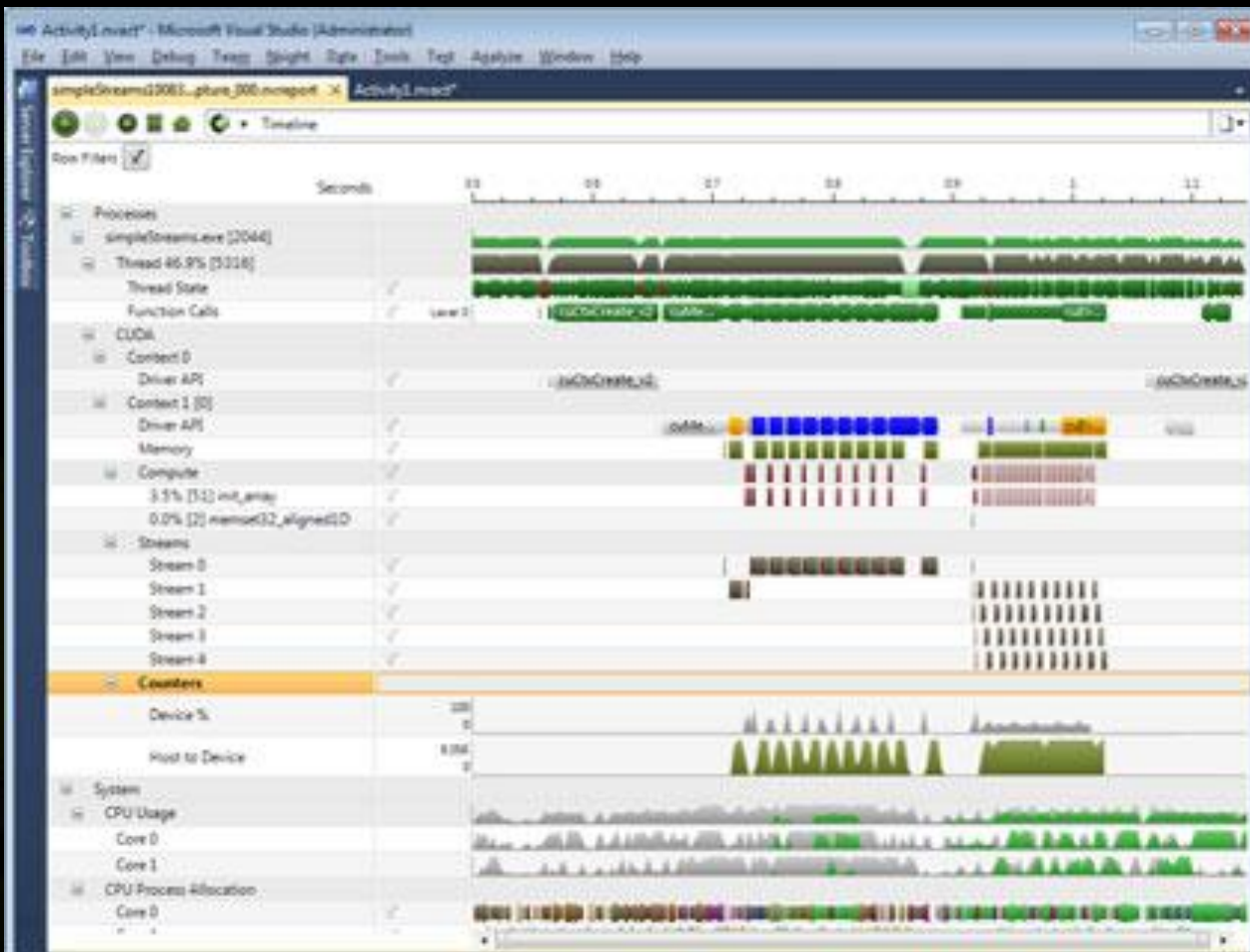
File Session View Options Window Help

Sessions: Session1, Device_0, Context_0 [CUDA]

Profiler Output | Summary Table | GPU Time Height Plot | Kernel Table | GPU Time Width Plot | Memcopy Table | Api Trace View

	GPU Timestamp	Method	GPU Time	CPU Time	Occupancy	grid size	block size	static st	register	mem tr	host me	branch Type:SM	divi Typ	instruc Type:S	gld coalesced Type:TPC
1	0	memcpy...	10,944	18,717						8192	Page...				
2	46,08	memcpy...	10,208	17,879						8192	Page...				
3	334,592	matrixMul	47,904	76,267	0,667	[2 4]	[16 16 1]	2084	13			113	1	1345	64
4	1177,34	matrixMul	45,664	70,959	0,667	[2 4]	[16 16 1]	2084	13			113	1	1345	64
5	1573,63	matrixMul	45,088	71,517	0,667	[2 4]	[16 16 1]	2084	13			113	1	1346	64
6	1962,75	matrixMul	46,304	72,076	0,667	[2 4]	[16 16 1]	2084	13			113	1	1346	64
7	2351,62	matrixMul	48,384	74,87	0,667	[2 4]	[16 16 1]	2084	13			113	1	1345	64
8	2746,62	matrixMul	46,336	73,752	0,667	[2 4]	[16 16 1]	2084	13			113	1	1346	64
9	3137,54	matrixMul	47,104	73,473	0,667	[2 4]	[16 16 1]	2084	13			113	1	1345	64
10	3527,68	matrixMul	45,632	71,797	0,667	[2 4]	[16 16 1]	2084	13			113	1	1345	64
11	3915,78	matrixMul	46,08	72,914	0,667	[2 4]	[16 16 1]	2084	13			113	1	1345	64
12	4305,66	matrixMul	47,2	74,032	0,667	[2 4]	[16 16 1]	2084	13			113	1	1345	64
13	4695,04	matrixMul	48,416	74,87	0,667	[2 4]	[16 16 1]	2084	13			113	1	1345	64
14	5086,46	matrixMul	44,832	70,959	0,667	[2 4]	[16 16 1]	2084	13			113	1	1345	64
15	5860,1	matrixMul	47,808	73,194	0,667	[2 4]	[16 16 1]	2084	13			113	1	1345	64
16	6256,9	matrixMul	48,672	75,987	0,667	[2 4]	[16 16 1]	2084	13			113	1	1345	64
17	6650,37	matrixMul	46,336	73,473	0,667	[2 4]	[16 16 1]	2084	13			113	1	1346	64
18	7043,07	matrixMul	47,712	74,032	0,667	[2 4]	[16 16 1]	2084	13			113	1	1345	64
19	7434,75	matrixMul	48,192	75,149	0,667	[2 4]	[16 16 1]	2084	13			113	1	1345	64
20	7826,18	matrixMul	47,296	73,473	0,667	[2 4]	[16 16 1]	2084	13			113	1	1346	64

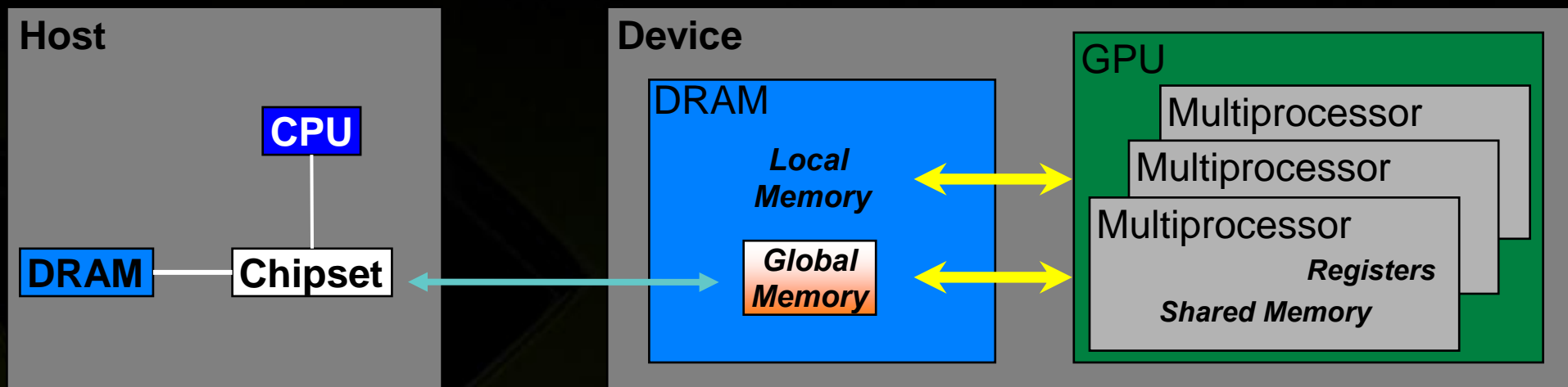
Parallel NSight 1.5 Professional



Memory Hierarchy



Managing Memory



L2 Cache L1 Cache



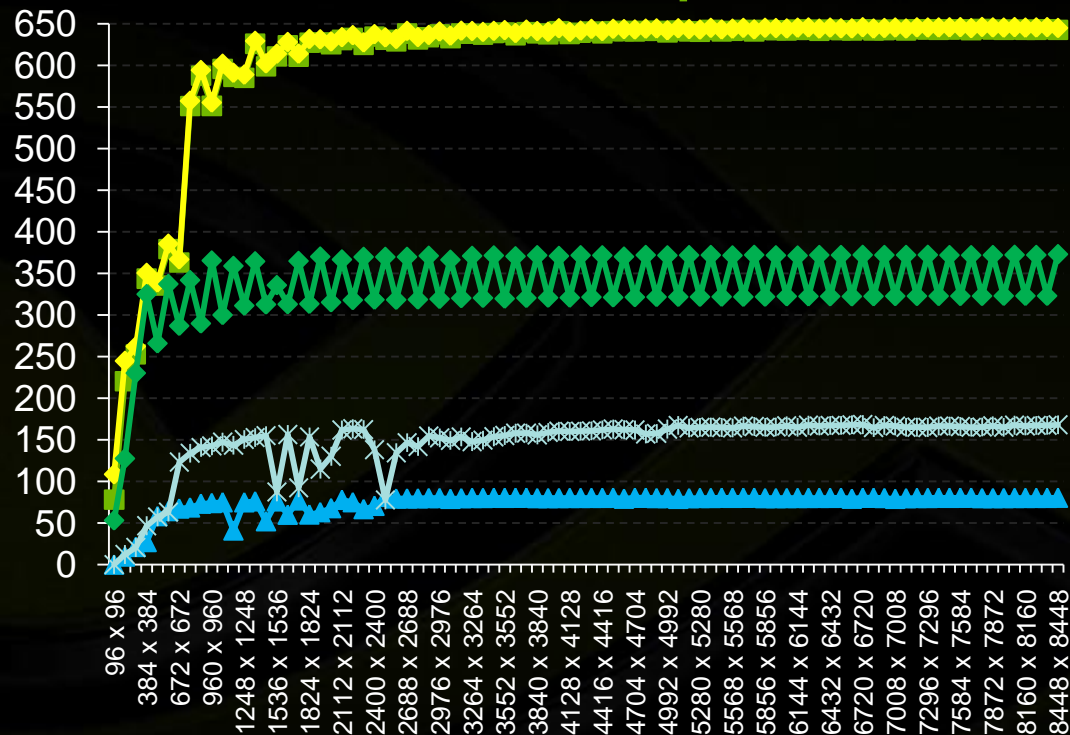
Bandwidth
Size

Matrix Size for Best CUBLAS3.2 Performance



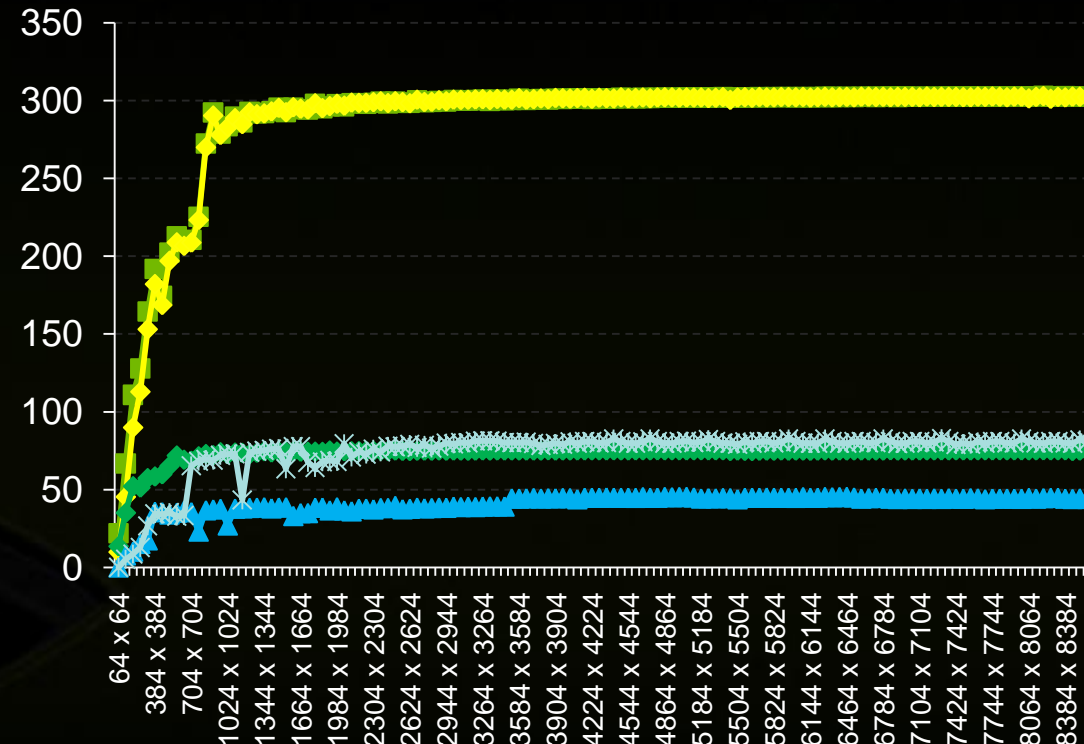
Gflops

SGEMM: Multiples of 96



Gflops

DGEMM: Multiples of 64



■ Tesla C2050 (ECC off)
 ◆ Tesla C2050 (ECC on)
 ◆ Tesla C1060
 ▲ MKL 4 Threads
 ✱ MKL 8 Threads

cuBLAS 3.2: NVIDIA Tesla C1060, Tesla C2050 (Fermi)

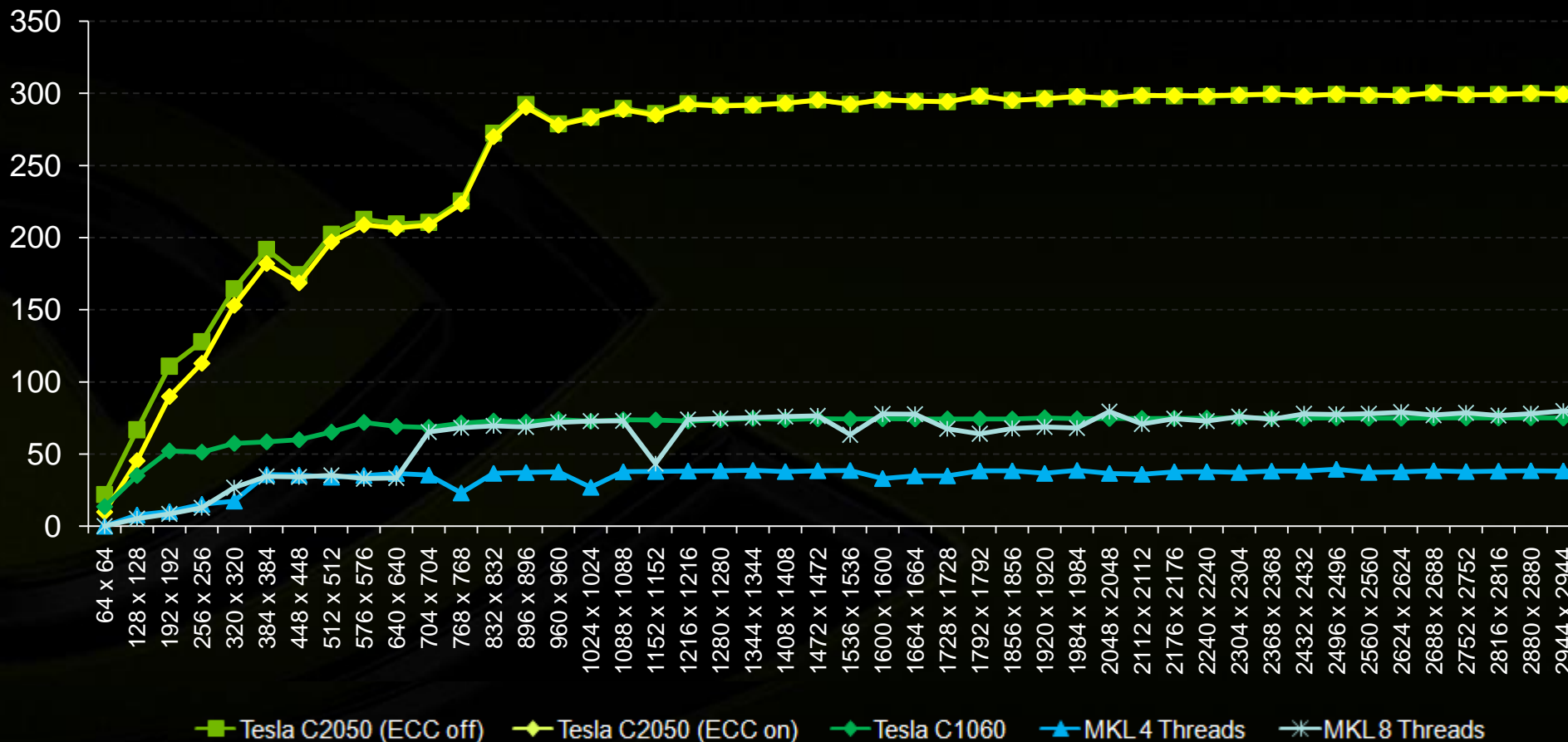
MKL 10.2.4.32: Quad-Core Intel Xeon 5550, 2.67 GHz

cuBLAS level III



Gflops

DGEMM: Multiples of 64



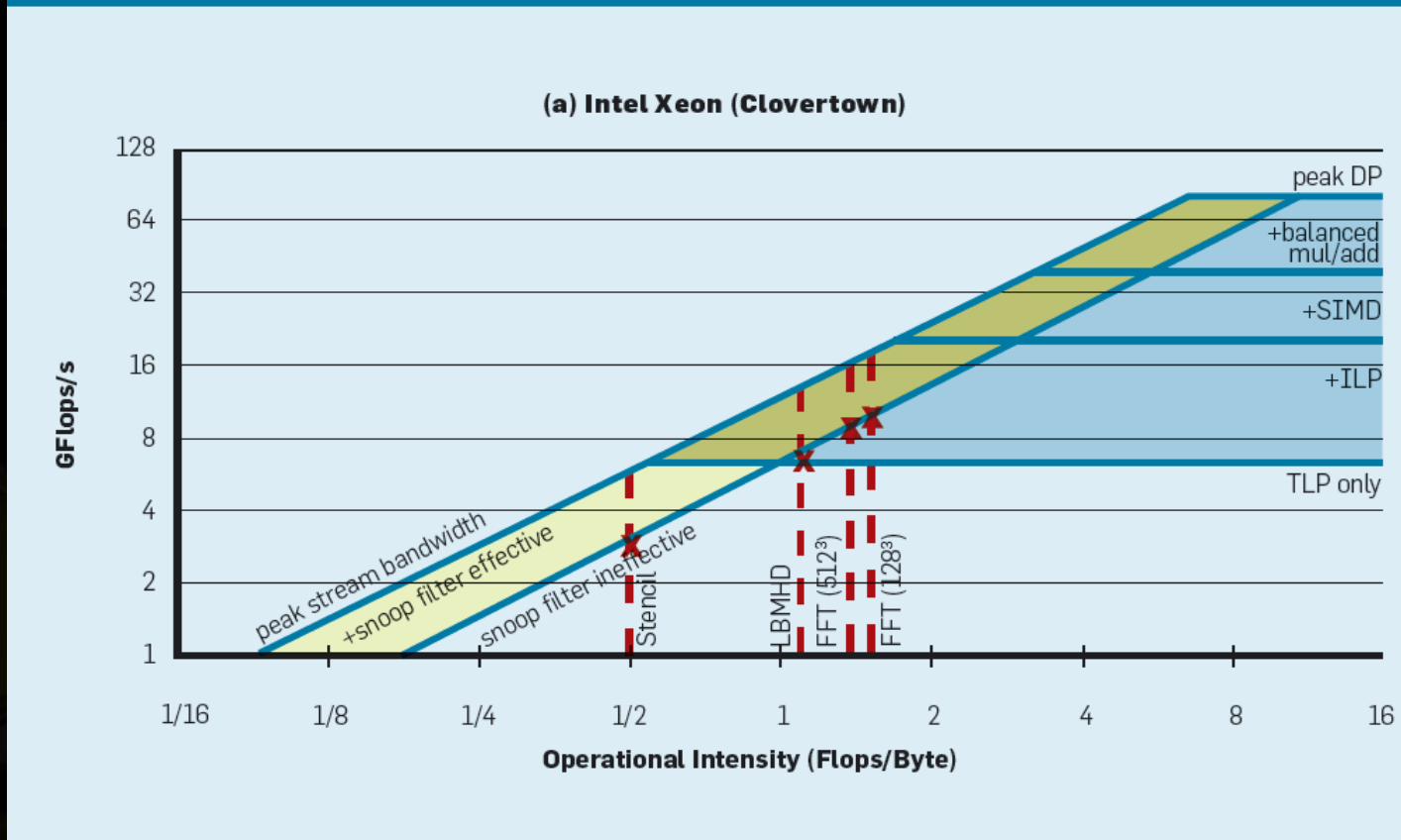
cuBLAS 3.2: NVIDIA Tesla C1060, Tesla C2050 (Fermi)

MKL 10.2.4.32: Quad-Core Intel Xeon 5550, 2.67 GHz

Roofline Analysis (Arithmetic Intensity)



Figure 3a–3c: Roofline model for Intel Xeon, AMD Opteron X4, and IBM Cell.



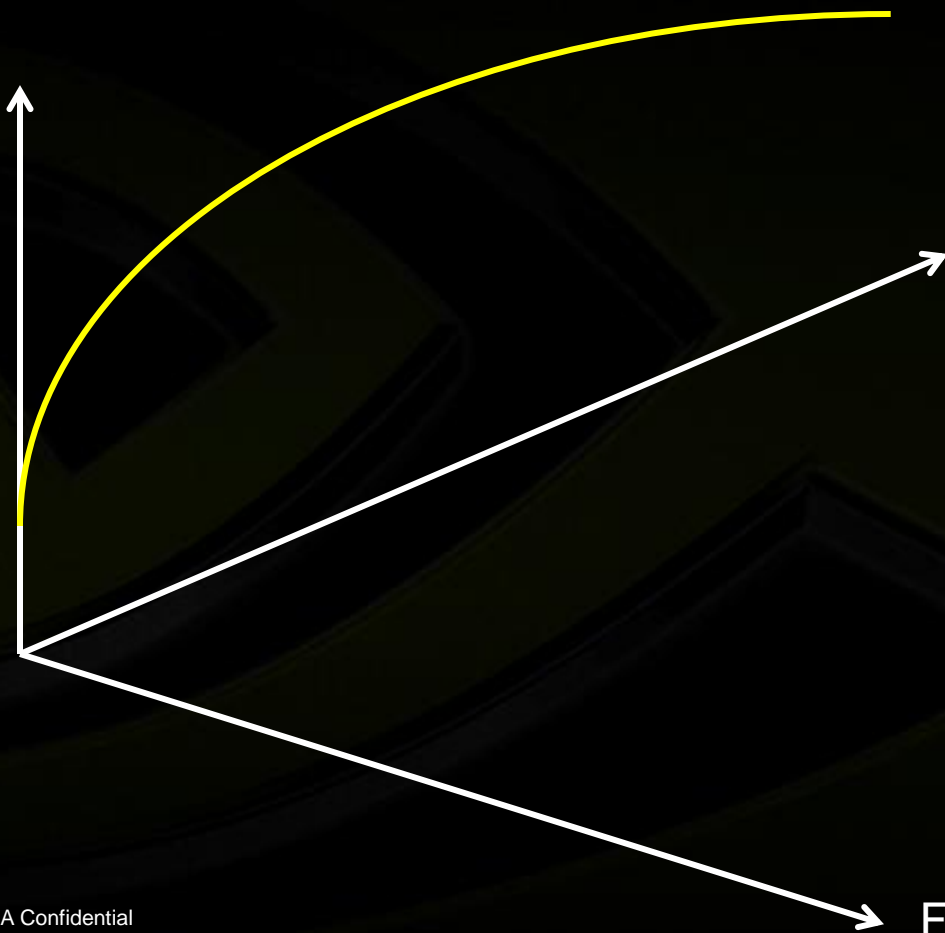
Samuel Williams, Andrew Waterman, and David Patterson,
Roofline: An Insightful Visual Performance Model for Multicore Architectures

Perf. on CUDA Application



Gflops

Bandwidth of PCI-e slot



Ns

Bandwidth of Global Memory

F/B

Tips for Optimization

- **Consider Algorithm for parallel (naïve algorithms will be good)**
- **Consider Occupancy (SIMT)**
- **Consider Memory Bottleneck**

More Information for CUDA Optimization

- **CUDA Zone**

<http://www.nvidia.com/CUDA>

- **Developer Zone**

<http://developer.nvidia.com>

- **GTC 2010 contents**

<http://www.nvidia.com/gtc2010-content>

- **쿠다 카페 (CUDA café in Korea)**

<http://cafe.daum.net/KCUG>



Thanks

Hyungon Ryu
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